Strategic Money and Credit Ledgers

Markus K. Brunnermeier and Jonathan Payne*

21 March 2024

Abstract

There is intense competition in the provision of new private currencies and settlement ledgers. We study which providers will prevail and how effectively the emergent market structure will enhance credit contract enforcement. If the economy has a sufficiently dominant tech platform, then the platform will "back" the settlement ledger, which enables it to incentivize lenders to coordinate on enforcing each others' contracts. It does so by threatening exclusion from both the payment and trading systems. Without a dominant tech platform, a private common ledger does not improve contract enforcement. Ledger provision tends to be a natural monopoly.

Keywords: Ledgers, currency competition, private currencies, "Industrial Organization of Money", smart contracts, platforms, Fintech.

*Brunnermeier: Princeton, Department of Economics, Bendheim Center for Finance, markus@princeton.edu. Payne: Princeton, Department of Economics, Bendheim Center for Finance, jepayne@princeton.edu. We are grateful to our discussant Charlie Kahn. We thank Mark Walker for outstanding research assistance. We are also grateful for comments from Joseph Abadi, Viral Acharya, Mark Aguiar, Jonathan Chui, Mikhail Golosov, Hanna Halaburda, Charlie Kahn, Nobu Kiyotaki, Thorsten V. Koeppl, Ye Li, Abdoulaye Ndiaye, Guillermo Ordoñez, Cecilia Parlatore, Laura Veldkamp, and Chaojun Wang.

1 Introduction

There are many attempts to introduce new currencies and settlement ledgers. Large tech platforms have started to offer their own tokens, payment services, and settlement systems (e.g. Alibaba, WeChat, Meta, Amazon). At the same time, large supply chains are moving payments and contracting onto shared ledgers (e.g. Corning, Emerson, Hayward). Communities have developed decentralized ledgers with automated contract enforcement (e.g. Ethereum, Solana). These changes raise important questions about how the provision of private settlement and currency ledgers impact the macroeconomy. Which institutions will provide settlement ledgers in an unregulated economy? Can new ledger providers enlarge the contracting space so that new credit contracts are possible? Will consumers benefit from such arrangements or will intermediaries gain market power? How should regulators respond? To address these questions, we build a general equilibrium production model with private control of settlement assets and ledgers.

We start with a three-period real model and then extend it to an infinite horizon monetary macro-model. In our three-period real model, there are producers who issue IOUs to purchase inputs in the initial period and then trade their output in one of the later periods. The economy has two trading technologies: a non-intermediated public marketplace and a private platform that is controlled by a profit maximizing platform that charges markups. There are also two payment technologies for settling trades: private "spot" exchanges and payment through a centralized ledger that can record and execute contracts. This environment can be thought of as having a ledger technology as in Kocherlakota (1998) but (i) without a benevolent planner organizing the ledger and (ii) with an outside payment option so that agents need to be incentivized to use the ledger. So instead of studying the planner problem, we study how successfully large private intermediaries are able to exploit the ledger technology.

We show that integration between the settlement ledger and platform trading technology is required to maximize contract enforcement. Having a common settlement ledger in the economy only leads to IOU repayment if the platform forces agents to pay through ledger when using their trading technology. Otherwise, all agents use spot trades and default. In other words, having a platform doesn't lead to contract enforcement unless it is connected to the ledger. Why? When producers trade before contract settlement, then the platform needs access to the settlement technology to be able to seize sales revenue and repay debtors. When producers trade after contracts are settled, then the platform needs the contract information on the ledger to threaten to exclude defaulting agents and incentivize repayment. Ultimately, a common settlement is only a powerful contracting technology if agents are incentivized to use the ledger — it needs to be "backed" by its usefulness on the trading platform in the economy.

The emergent market structure in an unregulated economy depends upon the fraction of trade controlled by the platform. A dominant platform will bundle the provision of a trading technology and a settlement ledger and use their market power to extract the maximum markup at which contract repayment in incentive compatible. Without a dominant platform, no common settlement ledger is created in the economy and no credit contracts are extended. Why? The platform faces the trade-off that charging higher markups increases profits but also makes it harder for the platform to back the settlement ledger because exclusion from platform trade is less costly for the agents. This means the platform finds incentivizing contract repayment too difficult unless they control a large fraction of trade. In this sense, there is a "natural" monopoly in contract enforcement.

In Section 3 we extend our analysis to an infinite horizon macroeconomic model where projects have flexible size, agents chose where to trade goods, agents need to save resources through financial intermediaries, and payments are settled using financial assets (IOUs on the ledger or money). We use this dynamic model to study feedback in the payment and trading systems. When agents can only receive revenue in financial assets recorded on the private ledger, then agents are locked into repayment. When agents can also receive revenue using money, then there is always an equilibrium in which financial intermediaries secretly allow producers to secretly store their sales revenue with them after defaulting on loans from other financial intermediaries.

The ledger operating platform can choose to eliminate this equilibrium by threatening to punish financial intermediaries that do not cooperate on contract enforcement by seizing their token holdings and excluding them from the ledger. Interestingly, the platform markup also affects the equilibrium interest rate. When markups are low more trades occur on the platform and more agents save in form of ledger IOUs which increases the loan supply. By contrast, when markups are high agents tend to hold cash until they want to consume, which limits loan supply and translates into a higher equilibrium interest rate. The higher interest rate partially offsets the markup disincentive to trade through the platform and so allows the platform to charge high markups without significantly losing customers.

Finally, we conclude our analysis by contrasting two policy responses: (i) regulating competition between multiple private platforms each providing their own — possibly interoperable — trading and settlement technologies and (ii) the provision of a public interoperable ledger. We show that competing private platforms that bundle ledger and trading technologies will cooperate on contract enforcement so long as the gap between their respective trading technologies is not too large and financial frictions do not prevent the less efficient platform from committing to pay the more efficient platform. Otherwise, a dominant platform emerges that attracts more trading and extracts higher rents. We show that introducing a public ledger resolves contracting problems in the economy if the government forces platforms to adopt the ledger but potentially incentivizes the platform to create a "black-market" with its own private tokens if the government does not force adoption of the public ledger. This is because previously the government was providing the unmonitored outside cash option and the platform was providing the contract enforcement where as no the platform is the one that can offer the hidden side-trading option.

Literature Review. Our paper is related to several branches the research, including on the literature concerning the role of ledgers and settlement assets in organizing trading systems. Aiyagari and Wallace (1991) and Kocherlakota (1998) study how a planner can increase the contracting space by updating a common ledger with trading histories. Freeman (1996b,a) studies how the choice of settlement asset creates or mitigates trading frictions in the currency market. Our model shares many features with these papers. However, we consider an environment where a private, profit-maximizing agent controls the ledger. This brings an industrial organization perspective to the literature on ledgers and settlement assets. In Brunnermeier and Payne (2023), we extend our model to study strategic information portability decisions in a contested market setting.

Second, we relate to the literature on currency competition (e.g. Hayek (1976), Kareken and Wallace (1981), Brunnermeier and Sannikov (2019)). Formally, our dynamic model in section 3 expands on the two currency cash-in-advance model from Svensson (1985) and endogenizes currency demand using search and trading frictions in the tradition of the new monetarist literature (e.g. Lagos and Wright (2005), Lagos et al. (2017)).

Third, we relate to the growing field of digital currencies. Instead of focusing on decentralized digital currencies such as cryptocurrencies (e.g. Fernández-Villaverde and Sanches (2018), Benigno et al. (2019), Abadi and Brunnermeier (2018), Schilling and Uhlig (2019), Cong et al. (2021)) or on central bank digital currency (e.g. Fernández-Villaverde et al. (2020), Keister and Sanches (2019), Kahn et al. (2019)), we are part of a less developed literature studying centralized digital currencies supplied by private tech platforms (e.g. Chiu and Wong (2020), Cong et al. (2020), Ahnert et al. (2022)). We argue that what makes "digital" currency special is its connection to a digital ledger and the associated increased contracting space.

Fourth, we relate to the literature on endogenizing debt limits when future income is difficult to pledge. In our model, the ledger operator can incentivize the repayment of debt contracts by threatening to seize assets and exclude agents from using their ledger. In this sense, the type of payment technology used determines the collateralizability of future sales revenue relating to the emerging literature on "digital collateral", (e.g. Garber et al. (2021)). Having a centralized platform can resolve the contracting issues across the supply chain presented in Bigio (2023). It also relates to Kahn and van Oordt (2022), where money is programmable and thereby offers users a commitment technology that stores resources in an escrow account until the payment is automatically executed.

The presence of cash as an alternative unmonitored payment technology in our model potentially allows agents to circumvent contract enforcement though "side-payments", similar to in Jacklin (1987) and Farhi et al. (2009). Rishabh and Schäublin (2021) shows the empirical counterpart. They document that after an Indian fintech company disbursed loans "digitally collateralized" by future digital sales revenue, borrowers' non-cash revenue drops. In our paper, the ledger controller resolves this enforcement difficulty by incentivizing financial intermediaries to report defaulters to other intermediaries so that they can coordinate on enforcement. This is in contrast to the literature, which has focused on incentivizing debtors directly.

We structure the paper in the following way. Section 2 solves the three period version of the model with a monopoly ledger controller and exogenous platform demand. Section 3 extends the analysis to an infinite horizon macroeconomic model with endogenous agent trading decisions. Section 4 concludes.

2 Monopoly Ledger and Enforcement

In this section, we outline a three-period version of our model. We use this model to highlight why integration between a trading platform and a common ledger is necessary for enforcing uncollateralized contracts in the economy. Without a trading platform forcing agents to use the ledger, agents undertake private side trades and default on loans. Without access to a common ledger, the platform has neither the enforcement power to take resources to enforce contracts nor the information to punish defaulting agents with exclusion from trade. Without government intervention, the emergent market structure is the platform controlling both the trading and ledger technologies and extracting rents from the economy.

2.1 Environment

Time lasts for three periods: $t \in \{0, 1, 2\}$, where $t = 1$ and $t = 2$ are interpreted as the morning and evening of second period. There is an input good and a collection of consumption goods. The economy contains two types of agents: input good producers and consumption good producers.

Production and preferences: Agents have consumption needs at random times, which mimics features of the life-cycle structure in the infinite horizon model. Input good producers can supply input goods at $t = 0$ at linear disutility and get linear utility from consumption in period $t = 1$. Consumption good producers can issue IOUs to purchase 1 input goods at $t = 0$ and transform it into $z > 1$ output goods at $t = 1$ and can frictionlessly store goods between periods 1 and 2. With probability $1 - \beta$ they get linear utility from consuming goods at $t = 1$ and with probability β they get linear utility from consuming at $t = 2$. In both cases they do not get utility from consuming the goods they produced on their own but instead only from goods produced by other agents.

Trading and payment technologies: There are two trading technologies for connecting agents in the economy, indexed by $n \in \{o, p\}$. Trading technology $n = o$ is not controlled by anyone and is referred to as the **"open" public marketplace**. Trading technology $n = p$ is controlled by a profit maximizing organization, which we refer to as the **private platform**. Agents find trading opportunities randomly each period. With probability $1 - \eta$ they find an opportunity on the public marketplace and with probability η they find an opportunity on the private platform.[1] We endogenize η in Section 3. The platform charges a markup μ_t at time $t \in \{1, 2\}$ on the profit received by sellers using their trading technology.[2] We impose that all profits from markups

[1] In the two period model, it does not matter where agents trade at $t = 0$. However, in the dynamic model, it will make a significant difference because agents can be locked into a trading system.

[2] Because production has fixed size, it doesn't matter whether the platform charges a markup on revenue or profit. In this section, we impose a markup on profit for notational convenience.

collected by the platform at t are redistributed lump sum to the agents trading at t as dividends.

All trade are settled using goods (we defer the introduction of money and bills of exchange in Section 3). However, there are two payment technologies in the economy for settling goods trades, spot payments and ledger payments, indexed by $m \in \{s, l\}$. **Spot payments** (s) are not recorded and are settled immediately. **Ledger payments** (l) are organized and recorded on a centralized ledger and settled at the end of each period.

Assets, markets, timing, and frictions: There is one type of asset: loans written in the form of IOUs by the output producers that promise R units of goods in period $t = 1$ for each input good given in period $t = 0$. Asset and goods markets open with the following timing. At $t = 0$, input producers sell input goods to output producers in exchange for IOUs. All $t = 0$ trades use the ledger. At $t = 1$, (i) output producers whose consumption needs realize early find trading opportunities on either the public marketplace or the platform, choose a payment technology, and exchange goods one-for-one, and (ii) input producers, who hold IOUs, return to the output producers to claim IOUs, and (iii) settlement (or default) occurs. Similarly, at $t = 2$, the remaining output producers find trading opportunities on either the public marketplace or platform, choose a payment technology, and exchange goods one-for-one.

The economy has information and enforcement frictions. Agents have publicly verifiable identities and so can be located in at times 1 and 2. However, agents' actions are not publicly observable and contract enforcement is imperfect. If an agent defaults on an IOU, then the holder cannot recover any revenue through the legal system and the borrower leaves with ψz, where $\psi \leq 1$. We interpret $(1 - \psi)z$ as the deadweight cost of default and endogenize it as a production distortion in Section 3. We assume that $\psi z \geq z - 1$ so that the agents always want to default at the competitive (or greater) interest rate of 1. The ledger automatically uses revenue from ledger trades to settle contracts within the period but revenue from spot trades cannot be used.

2.2 Market Equilibrium

We now characterize the equilibrium for different potential exclusions from the trading and payment technologies.

Input producer problem: Input producers choose whether to provide input goods in exchange for IOUs. Let ϕ denote the probability that an output producer repays a loan. The IOU interest rate at which input producers are indifferent about providing 1 input goods in exchange for IOUs must compensate the input producers for default:

$$1 = \phi R \quad \Rightarrow \quad R = 1/\phi \tag{2.1}$$

so long as $\phi > 0$. Feasibility requires that $R \leq z$. So, no loans are made if $1/\phi > z$.

Output producer problem: The output good producer chooses input good purchases, whether or not to default on the IOU, and a payment technology for trade. Let ι_s be an indicator for whether the platform allows agents to use spot trade. Let ι_d be an indicator for whether the platform allows agents to trade after having defaulted on the IOU.

If the output good producer trades at $t = 1$ (before contract settlement), then contracts are only enforced if they pay using the ledger and so the payment is taken before they can default. In this case, the $1 - \eta$ fraction of agents finding trading opportunities on the public marketplace choose spot and default while the η fraction of agents trading on the platform can only default if the platform chooses $\iota_s = 1$ and allows them to use spot trading. So the fraction of defaults on $t = 1$ trades is:

$$1 - \eta + \eta\iota_s.$$

If the producer trades at $t = 2$ (after contract settlement), then proceeds from trade cannot be used to redeem the contract at $t = 1$ because trade will not take place until $t = 2$. Instead, it is the threat of exclusion from future trade that might incentivise repayment. Then, the agent repays the loan iff:

$$(\eta(1-\mu_2)+1-\eta)(z-R) \geq (\eta(1-\mu_2)\iota_d+(1-\eta))\psi z \tag{2.2}$$

where the LHS is the benefit from repaying the loan and having access to all markets while the RHS is the benefit from defaulting but potentially only being able to trade on public market market.

At time $t = 0$, the output producers start projects so long as the profit from the project is positive.

Definition 1. Given a markup policy (μ_1, μ_2), a competitive equilibrium is a collection of an interest rate, r, and a default decision satisfying equations (2.1) and (2.2).

2.3 Market Arrangements and Contract Enforcement

We now contrast different market structure arrangements: independent platforms and ledgers, a monopoly provider of a bundled platform and ledger, and multiple ledger providers.

Independent platform and ledger: Theorem 1 shows that, if the ledger and trading technology are provided by separate institutions, then all output good producers default unless the institutions cooperate on enforcement.

Theorem 1. *If the common settlement ledger and trading technology are operated independently by separate institutions and one of the institutions does not cooperate on exclusion, then all output good producers default and there is no production in the economy.*

Proof. See Appendix A. □

Why is cooperation between the ledger operator and the platform required to ensure contracts enforcement? If the ledger automatically enforces contracts but the platform allows agents to undertake spot payments regardless of whether they have defaulted, then all agents use spot trades and default. If the platform attempts to ensure default, then it needs cooperation from the ledger. For agents that trade before contract settlement, the platform needs to force agents to use the ledger so that revenue can be seized to fulfil the contract. But, if the ledger operator does not allow the ledger to be used for payment on the platform, then the platform cannot do this. For agents that trade after contract settlement, the platform needs to exclude defaulting agents. But, if the ledger does not share contract information, then the platform cannot do this. In this sense, regardless of the timing of trade and settlement, the platform needs access to the ledger to be able incentivise contract enforcement.

Platform providing the common settlement ledger: Theorem 2 considers the behaviour of a platform that provides both a trading technology and a common settlement ledger for the economy. In this case, the platform chooses the markups the markups (μ_1, μ_2) at $t = 1, 2$ and has full control both exclusion decisions (ι_s, ι_d). It chooses these variables to solve problem (2.3) below:

$$\max_{\iota_s,\iota_d,\mu_1,\mu_2} \left\{\eta\Big(\mu_1(1-\beta)\pi_1(\mathcal{D}^{(1)}) + \mu_2\beta(1-(1-\iota_d)\mathcal{D}^{(2)})\pi_2(\mathcal{D}^{(2)})\Big)\right\} \tag{2.3}$$

where $\mathcal{D}^{(1)}$ is the default outcome by output producers trading on the platform at $t = 1$ under ι_s and $\mathcal{D}^{(2)}$ is the default decision by output producers trading on the platform at $t = 2$, which satisfies the IC constraint (2.2) and so implicitly depends upon ι_d and μ_2. The function $\pi_1(\mathcal{D}^{(1)}) = (1-\mathcal{D}^{(1)})(z-R) + \mathcal{D}^{(1)}\psi z$ is the profit early traders make under spot payment exclusion decision ι_s, the function $\pi_2(\mathcal{D}^{(2)}) = (1-\mathcal{D}^{(2)})(z-R) + \mathcal{D}^{(2)}\psi z$ is the profit late traders make when they choose default, the probability of repayment is $\phi = (1-\beta)\eta(1-\mathcal{D}^{(1)}) + \beta(1-\mathcal{D}^{(2)})$, and $R = 1/\phi$.

Theorem 2. *Suppose that the trading platform controls the common settlement ledger for the economy as well as the trading technology. The platform always sets $\mu_1 = 1$.*

For sufficiently large η, the platform incentivizes contract enforcement by maximising exclusion $\iota_s = \iota_d = 0$ and charging the maximum markup that is incentive compatible with contract enforcement at $t = 2$:

$$\bar{\mu}_2 = 1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\psi z}{z - R} - 1\right) \tag{2.4}$$

For sufficiently low η, the platform does not set up a ledger and there is no trade in the economy.

Proof. See Appendix A. □

Theorem 2 shows that the dominance of the trading platform, as measured by η, characterizes whether the economy has a problem with monopoly "rent" extraction or a problem with credit "fragility". The intuition for this is the following. The platform derives profit from charging markups on trade. However, it also incentivizes late consumer repayment by threatening to exclude them from trade. This means that increasing the markup to increase profits will also decrease the severity of the exclusion punishment. Together, these forces mean that the maximum markup the platform can charge and maintain contract enforcement is given by (2.4). When η is high, the platform can maintain a positive markup while still incentivising no-default and so is willing to set up the ledger. However, when η is low, the platform would have to offer a negative markup (i.e. a subsidy) to make exclusion from trade sufficiently costly to incentivize no-default. In this case, it prefers to not set up the common ledger.

Competing ledgers: Finally, we consider whether the platform is going to provide the ledger in an unregulated equilibrium. Corollary shows that if an independent ledger operator competes with a platform bundling a ledger, then the platform will ensure that it's ledger is dominant. This is because any ledger in our economy is backed by platform trade and so the platform can choose which ledger is valued in the economy. In this sense, the platform is the "natural" provider of the common ledger for the economy.

Corollary 1. *Suppose that a platform provides a ledger and an independent operator provides an alternative ledger. Then in equilibrium, the platform takes all the profit.*

Proof. This follows immediately from the previous results. The platform can exclude the alternative ledger from use on its platform, in which case the alternative ledger is never used. □

We close this section by summarizing the key lessons from our stylised three-period model that we explore further in subsequent sections.

(i) **Ledgers are only useful if they are "backed":** The ledger record keeping technology is potentially very powerful in the economy but only if agents use it. This means that platforms controlling the trading technologies in the economy need to "back" the ledger by forcing agents to use it. Otherwise, introducing the ledger technology will not change the equilibrium. In this sense, "BigTech" platforms are more natural providers of currency ledgers and "FinTech" services. So, the dominance of Alibaba and WeChat in the Chinese payment system might reflect their underlying advantage in providing payments.

(ii) **Payment technology as a way to collateralize sales revenue:** In this economy, the type of payment technology matters for the collateralizability of future sales revenue. Trades settled using the common ledger can always be used for the repayment of contracts and so essentially act as digital "collateral" for borrowing. Trades not settled using the ledger are not automatically used for the repayment of contracts and so can only be used as collateral if the agents coordinate on reporting and excluding defaulting agents. In this sense, the model is set up so the platform can choose to what extent future sales revenue can be collateralized across the economy. We extend this in the monetary model in Section 3, where we allow collateralized sales revenue to be traded as "bills-of-exchange" that relax the cash-in-advance constraint.

(iii) **Monopoly rents or credit fragility:** When the platform is very dominant in the economy we see a key trade-off with having the platform provide a common ledger. On the one hand, the platform takes actions to get agents to cooperate on enforcing default by threatening to seize their resources and exclude them from the ledger. In this sense, we get a "fortuitous" alignment of incentives where the platform wants to force the agent to behave in the way that they would choose if they could agree to coordinate. On the other hand, the ledger controller has a monopoly over the provision of ledger services and so is able to extract rents by charging a high transaction fee.

When the platform is not very dominant in the economy, the alignment of incentives breaks down and the platform is unwilling to provide the common ledger for the economy. This is because the cost of making exclusion from the platform a sufficiently harsh punishment becomes too much for the platform to pay. In this sense, the uncollateralized credit equilibrium is "fragile"; a weak platform finds it too costly for the platform to set up a no-default ledger. We explore these tradeoffs in more detail in the dynamic general equilibrium model in Section 3.

(iv) **A natural monopoly dilemma:** We can also see that there is a type of natural monopoly force in this economy. The more trade that uses the ledger (the higher is η), the more easily the ledger controller can enforce contracts. For example, suppose that the minimum η for which the no-default incentive compatibility constraint holds and $\bar{\mu} > 0$ is greater than $1/2$. In this case, there is no way for multiple ledgers to operate in the economy and enforce contracts unless they cooperate on enforcement. In other words, one large ledger provider can better enforce contracts than a collection of non-cooperative smaller ledger providers. So, a regulator in this environment needs to find a way to get a monopoly ledger provider to behave more competitively or have multiple large ledgers compete on markups while coordinating on contract enforcement. We take up these questions in section 3.6.

3 Dynamic Model of Platform & Ledger Provision

The model in Section 2 illustrates the value of having a large trading platform providing a settlement ledger. However, the model has some important limitations. First, the platform can extract all producer profits (subject to incentivising no-default) without distorting production. Second, agent decisions about where to trade are exogenous even though platform trading is what generates ledger demand. Third, all payment is in terms of goods despite contemporary ledgers being digital.

In this section, we address these limitations by integrating the model from Section 2 into an infinite horizon macroeconomic model where projects have flexible size, agents chose where to trade goods, payments are settled using financial assets (money or IOUs), and agents need to save resources through financial intermediaries. This means there now two key endogenous prices: the real exchange rate between marketplaces and the spread between the return on IOUs and money. We use this dynamic model to understand feedback in the payment and trading systems. We show that agents can be locked into repayment when they can only receive revenue in financial assets recorded through the ledger system. More generally, we show that the willingness of the platform to enforce contracts depends upon agents having a low elasticity of substitution between trading platforms. Finally, we show that platform markup choices impact both the real exchange rate between the marketplaces and interest rate in the economy. High markups discourage trade on the platform, which increases money demand and decreases IOU demand leading to high interest rates in the economy.

3.1 Environment Changes

Time is discrete with infinite horizon. There is a collection of goods that can be used for production and consumption. The economy contains a continuum of agents, mutual funds, and a private platform that operates a ledger for the economy. There is a money asset provided by the government and an intra-period digital token created by ledger. Each period is divided into a morning subperiod when a goods market opens and an evening subperiod when asset market opens. Agents need money or tokens to trade in the morning subperiod, as in Svensson (1985) and Lagos and Wright (2005).

Production, preferences, and life-cycle: Each agent follows a "life-cycle" where they start as producers and then become consumers. An agent born in the morning of time t (at age 0) arrives without resources but with a technology to convert x_t goods at time t into $y_{t+1} = zx_t^{\alpha}$ goods at time $t+1$, where $z > 0$ is productivity and $\alpha \in (0,1)$. At $t+1$ (at age 1), they sell their goods, consume, and save. At time $t+2$ (at age 2), they consume again, and exit. Instead of having random linear consumption demand, agents of generation t smooth consumption and rank it according to $(1-\beta)u(c_{1,t+1}) + \beta u(c_{2,t+2})$, where $u(c) = \log(c)$, $c_{\tau,t}$ is consumption of goods produced by other agents at time t when the agent is age τ, and $\beta \in [0,1]$. Although β is no longer stochastic, it none-the-less continues to characterize the fraction of producer consumption that is undertaken after contract settlement.

Goods market and trading frictions: As before, there is both a public marketplace and a private platform for connecting buyers and sellers, indexed by $n \in \{o,p\}$ respectively. However, now agents must choose where to trade. In addition to the pecuniary benefits from trading, each time period, t, for each technology, $n \in \{o,p\}$, each agent, i, gets an idiosyncratic, independent amenity draw for trading on that platform.[3] The draw for agents of age τ is distributed according to $\zeta_{\tau,t}^{ni} \sim \log(\zeta_\tau^n) + \mathrm{Gu}(1/\gamma_\tau, -(1/\gamma_\tau)\mathcal{E})$, where Gu denotes the Gumbel distribution and $\mathcal{E}$ is the Euler–Mascheroni constant, ζ_τ^n is a technology specific component that characterizes the average service quality provided by the platform to sellers, and γ_τ is the elasticity of substitution. For convenience, we normalize $\zeta_\tau^0 = 1$ and denote $\zeta_\tau^1 = \zeta_\tau$. We do not impose a physical interpretation on the amenity values but they could be modeled as idiosyncratic search costs or good quality.[4] At age 0, in the morning

[3]We introduce idiosyncratic risk in order to avoid "bang-bang" solutions to the platform choice problem. Our model uses tools from the discrete choice literature. This is analogous to assuming a CES preference function across the trading technologies.

[4]For the cost interpretation, note that the Gumbel distribution takes values across the real line

subperiod, the agents observe their amenity shock for trading at age 0 and 1.[5] At age 2, in the morning subperiod, the agents observe their amenity shock for trading at age 2. Each morning subperiod, a competitive goods market opens on each trading technology amongst the sellers and buyers who chose to go to the market. As before, the platform charges a markup μ on sellers.

Payment technologies and currencies: Once again, there are two payment technologies in the economy: spot trade and ledger trade. However, now we introduce currencies and settlement using financial assets. Spot transactions are not recorded and are subject to a resource-in-advance constraint: a fraction κ of the payment must be made using goods and/or public money ("dollars") issued by the government. We let $\overline{M}_t$ denote government money supply at t and rebate seigniorage to the funds proportional to their wealth. The other payment technology is the digital ledger provided by the platform, which is not subject to a resource-in-advance constraint. During the morning market, the digital ledger creates tradable tokens backed by non-risky future asset revenue to be received in the future through the ledger. In essence, this means that agents can use future ledger income to purchase goods. So, these tokens can be interpreted as "bills-of-exchange" for revenue inside the ledger ecosystem. As in Section 2, agents will choose spot trade, even on the platform, if not restricted to do otherwise. So, we start by imposing that all trade on the public marketplace is spot-trade while the platform mandates that sellers can only accept tokens when trading on the private platform. In this sense, money is the currency for spot trade while "bills of exchange" are currency for ledger trade. We relax these restrictions when we consider the incentive compatibility constraint in subsection 3.4.

Funds: There is a continuum of competitive mutual funds that pool resources across

and so ζ_τ^{ni} would represent a normalized cost. For the good quality interpretation, observe that we can write the total utility of a buyer receives as: $\log(e^{\zeta_\tau^{ni}}\zeta_\tau^n c)$ and so $e^{\zeta_\tau^{ni}}\zeta_\tau^n$ is essentially scaling the utility that the buyer gets from the good they consume.

[5]We make the assumption that agents observe their amenity shock for age 1 at age 0 for mathematical convenience so that we can get a closed form solution to the agent problem.

agents and facilitate currency exchange.[6] On the asset side of a fund's balance sheet, a fund can make one-period loans to producers, purchase equity in the platform, and hold reserves of currency. On the liability side, the fund issues one-period deposits that allow agents to withdraw money or tokens. The deposit interest rate they receive depends on which currency they request. We assume that the fund faces a "money-in-advance" constraint that it must hold money for withdrawals. There are two types of funds: those that accept defaulting agents and those that do not.

Asset, markets, and frictions: The economy has the following interperiod assets: dollars issued by the government, deposits, producer loans, and shares issued by the platform. The ledger also creates intraperiod tokens that pay one good during ledger settlement at the end of the morning market. All markets are competitive. Following the monetary literature, it will be helpful to distinguish between "money-goods" traded in dollar transactions and "token-goods" traded in ledger transactions. Let P_t^m denote the units of money required to purchases a good in the public marketplace in the morning market. Let P_t^b denote the units of tokens required to purchase a good on the private platform in the morning market. Let E_t denote the nominal exchange rate in the evening market: the tokens required to purchase 1 dollar. We typically use platform-goods as the numeriare and use "real prices" to refer to prices in terms of platform-goods. We define the the real prices of money and bonds as $q_t^m := E_t/P_t^b$, $q_t^b := 1/P_t^b$. We define the real exchange rate between marketplace goods and platform goods as $\epsilon_t := E_t P_t^m / P_t^b$ and the real price of shares as q_t^s. Where appropriate, we use $R_{t,t+1}^m$, $R_{t,t+1}^b$, and $R_{t,t+1}^s$ to denote the real return on holding money, bonds, and equity shares between t and $t+1$. We refer to money as the currency on the public market place and tokens backed by ledger assets as the asset on the private platform. We let $R_{t,t+1}^{bn}$ denote the effective real borrowing rate when the agent requests the loan in the medium of exchange on market n and let $R_{t,t+1}^{dn}$ denote the effective real deposit return set by the fund when agents withdraw deposits in the medium of exchange on

[6]Having the mutual funds simplify asset pricing. An equivalent structure would be to have a "Lucas family" that pools resources together but penalizes agents based on where they choose to trade or what type of currency they bring back to family.

market n.

The environment has the same information frictions as Section 2.1 except for two differences. First, default no longer has an exogenous deadweight loss. Instead, if an agent defaults on an IOU, then the holder can recover a fraction $\chi \in [0, 1)$ of the input good and the producer keeps the rest of their production. Second, the platform now excludes the funds accepting defaulting agents from the ledger technology rather than the individual agents.

Timing: Each period is divided into a morning and an evening sub-period. In the morning new IOUs are issued and goods are traded. In the evening, there is a secondary market for money and financial securities. The timing in the morning market is the following:

(i) Age 0 agents are born and start the period without any assets or inventory. They issue bonds that are discounted by the funds into the currency they need to use to buy input goods. Age 1 agents start the period with inventory and a payment type that they have chosen to accept. Age 2 agents start the period with holdings of deposits in the funds.

(ii) Goods markets open on each trading technology. Agents withdraw their wealth from the fund in the required currency (money or tokens). Agents trading in a particular currency participate in a competitive goods market.[7]

(iii) At the close of the goods market, producers repay loans (or default and face potential punishment). IOUs due are settled by the ledger. Depositors who purchase goods consume and exit.

The timing in the afternoon market is the following:

(i) Age 1 agents deposit revenue with the fund.

[7]The search literature often studies models where pricing is determined through one-to-one matching and bargaining over prices. Throughout this paper, we instead consider segmented competitive markets. We believe this is a closer approximation to the markets we are studying, especially in later sections when we model trade taking place on platforms such as Amazon or Alibaba.

(ii) The currency and asset markets open. Funds choose their asset portfolio for the next period, including their currency reserves.

3.2 Comparison to Other Models

Our environment attempts to nest or echo canonical models with currency and settlement frictions. We intentionally do not try to offer a novel theory of money or ledger demand. Instead, we are attempting to understand how large players in the economy strategically supply ledger technologies into an economy. To help make this clear, we discuss how this model relates to canonical models in the literature and why we have made particular deviations.

(i) *Cash-in-advance models (e.g. Svensson (1985), Lucas Jr and Stokey (1985)):* The timing of trade and construction of the "synthetic" real exchange rate between the two segmented markets is taken from the two-currency cash-in-advance model proposed by Svensson (1985). The difference in our set up is that the cash-in-advance constraint is only relevant on the public marketplace because the platform allows trade using digital tokens backed future guaranteed revenue. In this sense, trade on the platform is settled using digital bills of exchange that are automatically enforced at the end of the morning market. This is similar to the existence of both cash-good trades and credit-good trades in Lucas Jr and Stokey (1985) However, in our model, the creation of bills of exchange is not exogenous. Instead, the platform endogenously sets up trading rules to facilitate the creation of bills-of-exchange or credit-good trades in order to maximize their markup profit.

(ii) *Money search models (e.g. Lagos and Wright (2005)):* Like in Lagos and Wright (2005), we adopt a morning-evening subperiod structure where there are search frictions in the morning market that necessitate holding money or having access to a currency ledger. Unlike many papers in this literature, for simplicity, we abstract from bargaining between agents and instead consider segmented competitive markets. We also take the view that digital money trades occur with access to a currency ledger are so are necessarily monitored trades rather than anonymous trades in a decentralized markets. For this reason, we focus on how large institutions monitoring trades might supply digital currency ledgers rather than on how demand for digital money is different to demand for other monies.

(iii) *Social planner ledger provision* (e.g. Aiyagari and Wallace (1991) and Kocherlakota (1998)): Like in Kocherlakota (1998), we focus on how the introduction of a common record keeping technology can change equilibrium allocations. We have two main points of departure from Kocherlakota (1998): (i) we consider a ledger provided by a profit maximising platform and (ii) agents choose whether to use the ledger or an outside payment technology. If we took out spot trade and treated the platform as benevolent agent, then we would get back the results in Kocherlakota (1998).

(iv) *OLG models with financial assets* (e.g. Samuelson (1958), Diamond (1965)): Our model nests a classic OLG environment in which agents need an asset that they can buy when they are young and sell when they are old. In our environment, there are two assets available for storage that are differentiated by their usefulness in trade. In this sense, the financial assets in our model have both a role for storage and as a medium of exchange. Like in these models, the risk free rate in our model will end up being distorted. However, unlike in these models, it is the strategic behaviour of the platform controlling trade that leads to the interest rate distortion.

3.3 Market Equilibrium Without Default

In this subsection, we characterize equilibrium with agent choice about where to trade. We first characterize the buyer and seller problems under no-default. We then characterize market prices and show how platform markups affect general equilibrium. We use our characterization to study how the platform decisions interact with general equilibrium. In the next section, we return to the difficulties of loan enforcement and characterize the incentive compatibility constraint for no-default.

3.3.1 Agent Problem

We consider the problem for an agent in the generation born at time $t \geq 2$ and use the following terminology. Let n_τ denote the agent's choice of goods trading technology at age τ and let $\underline{\mathbf{n}}_\tau = (n_s : 0 \leq s \leq \tau)$ denote the history of technology choices up to age τ. We refer to money as the currency on the public market place and tokens backed by ledger assets as the asset on the private platform. Let ϵ_t^n denote the real exchange rate between goods on trading technology $n \in \{m, p\}$ and goods on the private platform if agents could access the afternoon asset market in the morning (and so is ϵ_t if $n = m$ and 1 if $n = p$).

We now set up the budget constraints. Consider the agent at age 0. The agent chooses where to purchase input goods, n_0, and where to sell output goods, n_1. If the agent purchase $x_{0,t}$ input goods on trading technology n_0, then they must issue $\epsilon_t^{n_0} x_{0,t}$ bonds.

At age 1 the agent then produces $y_{1,t+1} = z(x_{0,t+1})^\alpha$ goods, consumes $c_{1,t+1}$ goods, repays the loan, and deposits $d_{1,t+1}$ to a fund. If the agent sells on the private platform, $n_1 = 1$, then their revenue in platform goods is $(1 - \mu_{t+1}^{n_1}) y_{1,t+1}$. If the agent sells on the public marketplace, $n_0 = 0$, then their revenue in units of platform goods is $\epsilon_t y_{t+1}$. Thus, their budget constraint at $t = 1$ in real terms when they buy inputs on n_0 and sell on n_1 is:

$$d_{1,t+1} \leq (1 - \mu_{t+1}^{n_1}) \epsilon_{t+1}^{n_1} \left(z(x_{0,t})^\alpha - c_{1,t+1} \right) - \epsilon_t^{n_0} R_{t,t+1}^{bn_0} x_{0,t} \tag{3.1}$$

Finally, at 2, the agent chooses a marketplace on which to consume, n_2. If they choose n_2, then the real value of their withdraws is $R_{t+2}^{d,n_2} d_{1,t+1}$ and so the agent faces the budget constraint:

$$\epsilon_t^{n_2} c_{2,t+2} \leq R_{t+1,t+2}^{d,n_2} d_{1,t+1} \tag{3.2}$$

Taking price processes as given, at age 0, an agent in generation t solves problem (3.3) below:

$$\begin{aligned} &\mathbb{E}_t\Big[\max_{x_0,c_1,c_2,d_1,\underline{\mathbf{n}}}\Big\{\zeta_{0,t}^{n_0}+\zeta_{1,t+1}^{n_1}+(1-\beta)u(c_{1,t+1})+\beta(\zeta_{2,t+2}^{n_2}+u(c_{2,t+2})\Big\}\Big] \\ &s.t. \quad (3.1),(3.2). \end{aligned} \tag{3.3}$$

where $\zeta_{\tau,t}^{n}$ is the idiosyncratic amenity of using trading technology n at age τ.

Theorem 3. *An agent choosing trading technologies* $\underline{\mathbf{n}}_2=(n_0,n_1,n_2)$*, undertakes production:*

$$\begin{aligned} x_{0,t} &= \left(\frac{\alpha z(1-\mu_{t+1}^{n_0})}{\epsilon_t^{n_0}R_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad\qquad y_{1,t+1}=z\left(\frac{\alpha z(1-\mu_{t+1}^{n_0})}{\epsilon_t^{n_0}R_{t,t+1}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}}, \\ \pi_{1,t+1} &= \left(\frac{\epsilon_{t+1}^{n_1}\alpha z(1-\mu_{t+1}^{n_0})}{(\epsilon_t^{n_0}R_{t,t+1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right). \end{aligned} \tag{3.4}$$

and chooses consumption and saving:

$$c_{1,t+1}=\frac{(1-\beta)\pi_{1,t+1}}{\epsilon_{t+1}^{n_2}}, \qquad d_{1,t+1}=\beta\pi_{1,t+1}, \qquad c_{2,t+2}=\frac{R_{t+1,t+2}^{dn_2}\beta\pi_{1,t+1}}{\epsilon_{t+2}^{n_2}}. \tag{3.5}$$

The fraction of producers and consumers respectively choosing trading technologies n_0*,* n_1*, and* n_2 *are:*

$$\eta_{0,t}^{n_0}=\frac{\left(\zeta_0^{n_0}(\epsilon_t^{n_0}R_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}(\epsilon_t^{n_0'}R_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \tag{3.6}$$

$$\eta_{1,t+1}^{n_1}=\frac{\left(\zeta_1^{n_1}\,(\epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1}))^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_{t+1}^{n_1'}(1-\mu_{t+1}^{n_1'})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}} \tag{3.7}$$

$$\eta_{2,t+2}^{n_2}=\frac{\left(\zeta_2^{n_2}R_{t+1,t+2}^{dn_2}/\epsilon_{t+2}^{n_2}\right)^{\gamma_2}}{\sum_{\mathcal{L}'}\left(\zeta_2^{n_2'}R_{t+1,t+2}^{dn_2'}/\epsilon_{t+2}^{n_2'}\right)^{\gamma_2}} \tag{3.8}$$

Proof. See Appendix B. □

From Theorem 3, we can see the expected relationships between the variables. Holding everything else constant, an increase in the private platform trading advantage, $\uparrow \zeta_\tau^p/\zeta_\tau^o$ leads to more agents using the private platform at each age. An increase in the effective cost of borrowing on a trading technology, $\uparrow R_{t,t+1}^{bn_0}$, leads to fewer producers purchasing input goods there. Likewise, an increase in the effective price of goods on a trading technology, $\uparrow \epsilon_{t+2}^{n_2}$, leads to fewer agents purchasing consumption goods there. Finally, an increase in the effective price of goods (net of fees), $\uparrow \epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1})$, leads to more agents selling on the platform at age 1.

3.3.2 Fund Problem

In the evening of each period, t, the funds take deposits, D_t, from agents. They then purchase money, M_t, (to back deposit withdrawals and finance issuance of loans in dollars), purchase bonds, B_t, and purchase shares in the platform. So, their budget constraint at t is:

$$q_t^m M_t + q_t^b B_t + q_t^s S_t \leq D_t. \tag{3.9}$$

In the morning of $t+1$, depositors trading on the public marketplace ($n=m$) withdraw money and depositors trading on the private platform ($n=p$) take tokens backed by the market value of the remaining assets in the fund. The fund offers depositors withdrawing in money the return on money and the other depositors the return on assets that can be used in exchange on the ledger. So the "money-in-advance" constraint on the fund is:

$$\kappa\eta_{2,t+1}^m R_{t,t+1}^{dm} D_t \leq q_{t+1}^m M_t \quad \Leftrightarrow \quad \kappa\eta_{2,t+1}^m D_t \leq q_t^m M_t \tag{3.10}$$

Lemma 1 states fund sets returns to offset the opportunity cost of having to hold money or bonds.

Lemma 1. *If the money-in-advance constraint binds, then the borrowing rate and* *deposit rate faced by agents using trading technology n are give by:*

$$R^{bn}_{t,t+1} = \frac{R^b_{t-1,t}}{(1-\kappa)R^b_{t-1,t} + \kappa R^n_{t-1,t}} R^b_{t,t+1}$$
$$R^{dn}_{t,t+1} = (1-\kappa)R^b_{t,t+1} + \kappa R^n_{t,t+1} \tag{3.11}$$

Proof. See Appendix B. □

3.3.3 Market Equilibrium

We now consider market equilibrium. We define equilibrium for an arbitrary sequence of markup policies although we will ultimately focus on the steady state limit with a fixed markup.

Definition 2. Given a sequence of ledger policies, $\underline{\boldsymbol{\mu}}$, a competitive equilibrium is a collection of price and return sequences, $(\underline{\mathbf{R}}^b, \underline{\mathbf{R}}^m, \underline{\mathbf{q}}^s)$, agent choice sequences, $(\{\underline{\boldsymbol{\eta}}_\tau\}_{\tau\le 2}, \underline{\boldsymbol{x}}, \underline{\boldsymbol{y}}, \underline{\boldsymbol{c}}_1, \underline{\boldsymbol{c}}_2)$, such that: (i) given prices, the agent choices solve optimization problem (3.3), (ii) given prices, the fund choices solve equation (3.11), and (iii) market clearing is satisfied for the goods market on each trading technology, the IOU market, the money market, and the equity market:

$$\begin{aligned}
\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}y^{(n_0,n)}_{1,t} &= \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}c^{(n_0,n)}_{1,t} \\
&\quad + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}, \qquad \forall n \in \{o,p\}, \\
D_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}d^{(n_0,n_1)}_{1,t}, \\
q^b_t B_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t x^{(n_0,n_1)}_{0,t}, \\
M_t &= \overline{M}_t, \\
S_t &= 1.
\end{aligned}$$

Theorem 4 characterizes equilibrium under some parametric restrictions on the elasticities of substitution for buyers at ages 0 and 2. Our environment is sufficiently tractable that we get a closed form expression for the real exchange rate ϵ_t. However, the expressions for $R^b_{t,t+1}$, $R^m_{t,t+1}$, and q^s_t are necessarily difference equations because previous agent choices discipline their current

Theorem 4. *Suppose that that* $\frac{1+\alpha\gamma_0}{1-\alpha} = 1 + \gamma_2$ *and* $(\zeta^n_0)^{\gamma_0} = (\zeta^n_2)^{\gamma_2}$. *Let the excess return on bonds over money required to satisfy the cash in advance constraint be denoted:*

$$\mathcal{S}_{t,t+1} = \frac{R^b_{t,t+1}}{(1-\kappa)R^b_{t,t+1} + \kappa R^m_{t,t+1}}$$

Then the real exchange rate, bond return, price of money, and equity price satisfy:

$$\epsilon_t = \left[\frac{\zeta_1^{\gamma_1}\left(1-\mu_t\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}}{\zeta_2^{\gamma_2}\mathcal{S}_{t-1,t}^{1+\gamma_2}}\right]^{\frac{1}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}} \tag{3.12}$$

$$R^m_{t,t+1} = \left(\frac{\eta^o_{2,t+1}D_{t+1} + X_{0,t+1}}{\eta^o_{2,t}D_t + X_{0,t}}\right)\left(\frac{\overline{M}_t}{\overline{M}_{t+1}}\right) \tag{3.13}$$

$$q^s_t = \frac{1}{R^b_{t,t+1}}\left(\pi^s_{t+1} + q^s_{t+1}\right) \tag{3.14}$$

$$\eta^p_{2,t}\beta\Pi_{1,t} = \sum_{n_1}\eta^{n_1}_{1,t+1}\left(\eta^o_{0,t}\epsilon_t\left(1+\frac{R^b_{t-1,t}}{R^m_{t-1,t}}\right)x^{(o,n_1)}_{0,t} + \eta^p_{0,t}x^{(p,n_1)}_{0,t}\right) + q^s_t \tag{3.15}$$

where $X_{0,t} := \sum_{n_1}\eta^o_{0,t}\eta^{n_1}_{1,t+1}\epsilon_t\mathcal{S}_{t-1,t}x^{(o,n_1)}_{0,t}$ *is aggregate input purchases on marketplace* o *at time* t, *and* $\Pi_{1,t} := \sum_{n_0,n_1}\eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}\pi^{(n_0,n_1)}_{1,t}$ *is aggregate profit at time* t.

Proof. See Appendix B. □

Evidently, the real exchange is decreasing in μ_t and increasing in the excess return on bonds over money required to satisfy the cash-in-advance constraint. This is because an increase in μ encourages producers to choose the public public marketplace while an increase in the opportunity cost of holding money for the public marketplace encourages buyers to choose the private platform.

The return on money is essentially the ratio of money demand growth to money supply growth. This implies that, in the steady state, we have the familiar formula that $R^m_{t,t+1} = 1/g_M$, where g^M is the growth rate of money. As is standard in "currency-in-advance" models, the environment has money neutrality in the sense that the level of money supply does not affect real variables. However, it does not have super-neutrality in the sense that the growth rate of money affects real variables by impacting agent borrowing decisions. The price of equity is given by future dividends discounted by the bond rate.

The final equation implicitly gives the bond interest rate in the economy. To help isolate forces, consider the steady state in which $\mu_t = \mu$, $\epsilon_t = \epsilon$, $R^b_{t,t+1} = R^b$, $R^m_{t,t+1}$, and $q^s_t = q^s$ are all constant. In addition, suppose that $\alpha = 0.5$ and the government follows a "Friedman rule" so that $\mathcal{S}_{t,t+1} = 1$. In this case, the interest rate satisfies:

$$R^b = \frac{\sum_{n_1} \eta_1^{n_1} \left(\alpha z(1-\mu^{n_1})\epsilon^{n_1}\right)^{\frac{1}{1-\alpha}} \left(2\eta_0^o \left(\epsilon\right)^{-\frac{\alpha}{1-\alpha}} + \eta^p\right) + \frac{\pi(R^b)^2}{R^b-1}}{\eta_2^p \beta \sum_{n_0,n_1} \eta_0^{n_0} \eta_1^{n_1} \left(\frac{\alpha z(1-\mu^{n_1})\epsilon^{n_1}}{(\epsilon^{n_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)}$$

which is a quadratic in R^b. The numerator reflects the demand for ledger assets by the fund while the denominator reflects the supply of deposits that can be used to purchase ledger assets, given the cash-in-advance constraint for trading on the public marketplace. Observe that a decrease in ζ_2 that leads to a decrease in η_2^p causes an increase in the loan rate. This is because a decrease in η_2^p means that more depositors withdraw money and so the fund has to hold more money. As a result the supply of fund deposits that can be used to purchase ledger assets decreases and so the return on ledger assets increases.

Now suppose we relax the Friedman so that $\mathcal{S}$ depends upon R^b and agent decisions about where to trade are impacted by R^b. In this case, the agent trading behaviour introduces feedback into the interest rate. A decrease in ζ_2 still pushes η_2^p down but this is partially offset by the increase in the bond rate which encourages agents to save into ledger assets and trade on the platform.

To help illustrate these forces, we plot the steady state equilibrium in Figure 1 as a function of the platform markup. The blue dashed line show the partial equilibrium allocations as μ varies when the interest rate is fixed at 3%. The black solid line shows general equilibrium when the equilibrium is allowed to vary. From the blue dashed lines we can see that a higher markup leads to fewer agents going to the platform, which ultimately decreases the real exchange rate (i.e. makes the public marketplace relatively cheaper). From the black solid lines we can see that, in general equilibrium, the loan rate, R^b, increases to encourage agents to hold ledger assets and come to the platform. Ultimately, this ends up causing the platform markups to have a much greater impact on output. In this sense, the platform trying to extract rents in the product market restricts the supply of credit in the loan market.

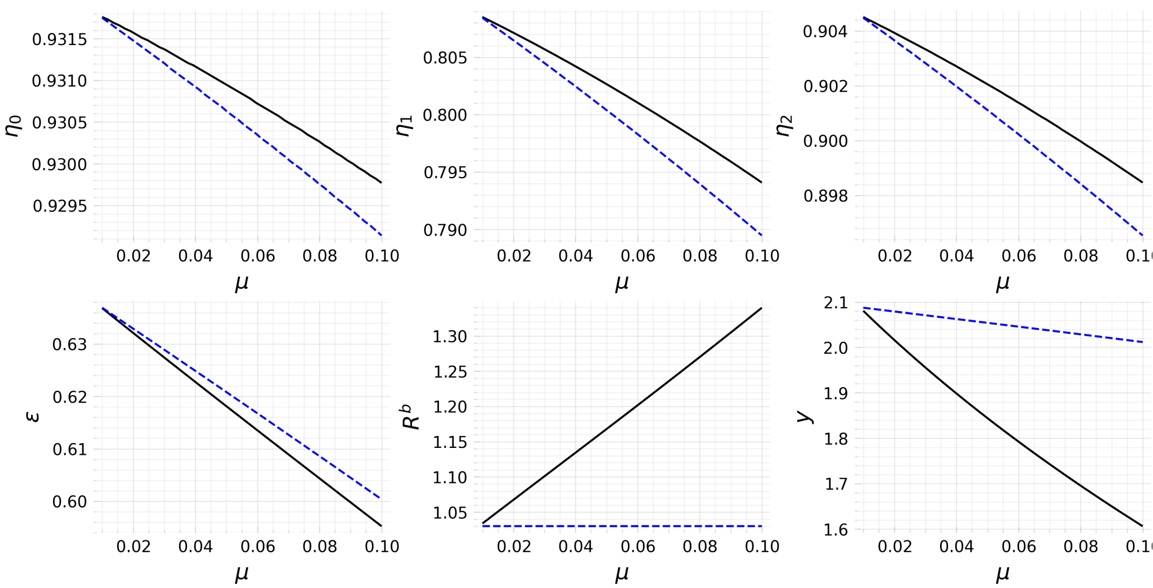

Figure 1: Equilibrium for $\mu \in [0, 0.1]$.

Other variables are $z = 1$, $\alpha = 0.45$, $\beta = 0.9$, $\gamma_1 = 1.9$, $\gamma_2 = 1.5$, $\zeta = 1.0$, and $\kappa = 0.1$.

3.4 Default and Incentive Compatibility

We now return to the question of incentive compatibility in the dynamic model. Funds that accept defaulting agents cannot use the payment ledger because as soon as they store wealth in the fund, their resources will be taken to fulfill contracts. So, if agents default and deposit into a fund, then they are restricted to the monetary system. The agent problems are very similar so we defer the details to Appendix B.

Theorem 5 states the main result: the incentive compatibility constraint on deterring agents from defaulting.

Theorem 5. *The incentive compatibility constraint is:*

$$\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} \geq \frac{\check{\pi}}{\pi} \tag{3.16}$$

where $\bar{\nu}_{2,t+2}/\check{\nu}_{2,t+2}$ is the relative benefit of having access to the platform at $t = 2$ and $\check{\pi}/\pi$ is the relative additional profit when the agent defaults:

$$\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} = \left(\frac{R_{t+1,t+2}/\epsilon_{t+2}}{\sum_{n_2} \left(\zeta^{n_2}_{t+2} R^{n_2}_{t+1,t+2}/\epsilon^{n_2}_{t+2} \right)^{\gamma_2}} \right)^{\beta\gamma_2}$$

$$\frac{\check{\pi}}{\pi} = \left(\frac{R^{bn_0}_{t,t+1}}{\chi} \right)^{\frac{\alpha}{1-\alpha}}$$

For $\chi > 0$, the incentive compatibility constraint will be satisfied for sufficiently large ζ_2.

Proof. See Appendix B. □

Contract enforcement is similar to in the three-period model. If agents trade using the platform, then they have to repay because the platform forces them to use the ledger. If they trade using the public marketplace, then they can trade using money, default and deposit into the shadow financial system. The IC constraint for non-default when trading on the public marketplace is given by equation (3.16). This is the analogue of equation (2.2) in the three-period model. It says that the benefit of having access to the platform at $t = 2$ needs to be greater than the additional profit gained by defaulting.

To help illuminate enforcement in the dynamic model, we consider a collection of special cases.

Case: $\beta = 1$ and no money. If there is no money in the economy and agents only value consumption at age 2, then the presence of a ledger is sufficient to incentivise repayment, regardless of what the platform does. Why? When $\beta = 1$, agent do not value consumption at $t = 1$ and do not engage in barter trade with other agents of their generation for perishable goods. Instead, they only trade goods to agents of generations in exchange for financial assets. If there is no money, then age 1 agents selling goods receive newly issued IOUs from the age 0 agents and old IOUs from the age 2 agents purchasing goods. Since all IOUs are on the ledger, there is no way for the age 1 agents to default, regardless of whether the platform excludes defaulting agents or not. In this sense, the agents are locked into the ledger payment system because they need a way to store wealth.

Case: $\beta = 1$ with money and ledgers. Now, suppose we introduce money into the model with $\beta = 1$. Now, agents have an outside payment option and the platform is necessary for ensuring contract enforcement in the economy. The have no need to consume at $t = 1$ and so all trade occurs after contract settlement. In this case, platform exclusion from purchasing consumption goods at $t = 2$ can discipline no-default.

Case: $\beta = 0$ with money and ledgers. In this case, exclusion from trade at $t = 2$ is no longer relevant. In order to get no-default, the platform has to ensure that the platform price is sufficiently high that the opportunity cost of not being able to sell on the platform is a sufficiently high punishment for no-default. If ζ_1 is low, then this requires a negative markup μ, running the risk of credit fragility as in Section 2.

3.5 Platform Problem (Sequential Formulation)

We can now write down the problem of a private platform. Suppose the economy starts with an initial collection of age 1 agents with loans and goods inventory $(b_{0,0}, y_{1,0})$ and a collection of age 0 agents with wealth $(a_{2,0})$ in a collection of funds. Given a belief about the interest rate processes, $\{\hat{R}^b, \hat{R}^m\}$, the platform chooses a sequence $\boldsymbol{\mu}$ to maximise their equity price by solving problem:

$$
\begin{aligned}
q_0^s = \max_{\boldsymbol{\mu}} & \left\{ \sum_{t=0}^{\infty} \hat{\xi}_{0,t} \pi_t^s \right\} \quad s.t. \\
& \pi_t^s = \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1} y_{1,t}^{(n_0,p)}, \quad t \geq 1 \\
& \pi_0^s = \mu_0 \eta_{1,0} y_{1,0} \\
& \text{Agent choices: } (3.4), (3.5), (3.6), (3.7), (3.8), \\
& \text{Equilibrium prices: } (3.12), (3.13), (3.14), (3.15)
\end{aligned}
$$

where $\hat{\xi}_{0,t} = \prod_{j=0}^{t} (\hat{R}_{j,j+1})^{-1}$ is the household SDF.

We defer the first order conditions for this problem to Appendix B.3. Figure 2 plots the numerical solution to the equilibrium for different values of buyer elasticity. Evidently, for low γ^b, the constraint on incentivising default is non-binding because it is easy for the platform to attract buyers. However, as γ^b increases, it becomes more difficult for the platform to attract buyers and so the constraint eventually binds. Once this occurs, the platform must significantly decrease ψ in order to attract enough buyers to achieve the no-default equilibrium. In this sense, buyer elasticity is the key variable for understanding to what extent platform rent extraction is disciplined by the platform needing to attract traders.

Credit Fragility: Figure 2 shows that for very large γ^b, the platform actually needs to sets a negative markup (i.e. a subsidy) in order to attract sufficiently many traders which allows it to enforce contracts. This means that the platform derives negative value from running a non-default ledger and so would not choose to ensure contract enforcement. In this sense, the uncollateralized credit equilibrium is “fragile”; strong competition from the dollar and public marketplace make it too costly for the platform to set up a no-default ledger. This implies that regulatory changes that increase agent ability to switch away from trading platforms would not necessarily be welfare improving.

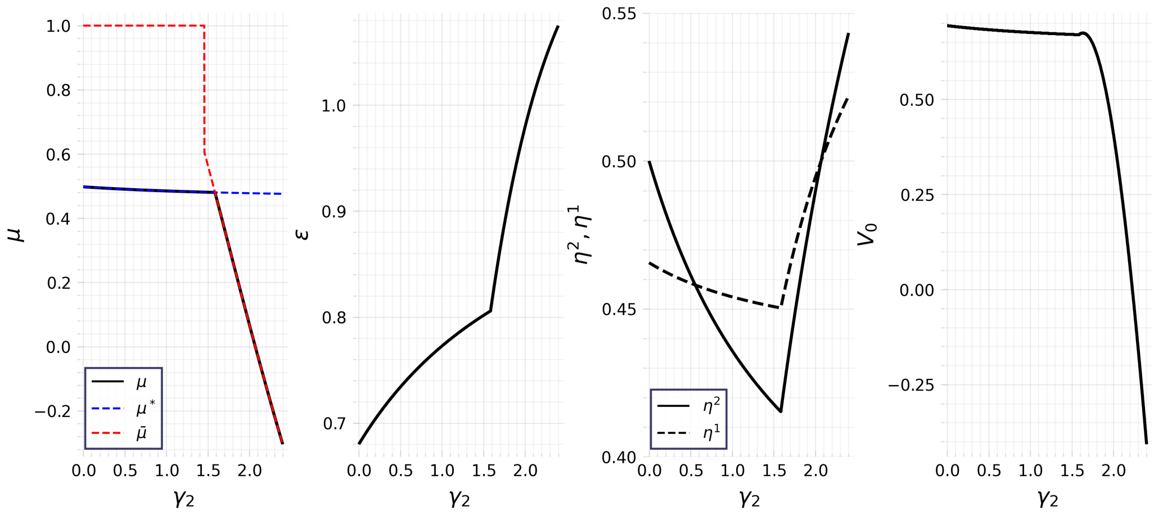

Figure 2: Steady state solution to platform problem for $\gamma_2 \in [0, 2]$.

Panel 1: The blue dashed line depicts the platform's markup choice, μ^* if they are not constrained by having to ensure no-default. The red dashed line depicts the minimum value $\bar{\mu}$ required to deter funds from defaulting. The black line depicts the markup for the equilibrium chosen by the platform, μ. Panel 2: shows the equilibrium real exchange rate under the platform's choice of μ. Panel 3: shows the fractions of buyers and sellers that choose platform 1 given μ. Panel 4: shows the time 0 value of a platforms that sets up a ledger and chooses the no-default equilibrium. Other variables are $\gamma_1 = 0.1$, $z = 1.0$, $\alpha = 0.45$, $\rho = 0.2$, $\lambda = 1.0$, $\zeta = 1$, $\kappa = 0.1$, and $\chi = 0.5$.

3.6 Discussion of Regulatory Options

We have shown that a tech platform will provide and "back" a common settlement ledger in an unregulated economy if they have a sufficiently dominant trading technology. This incentivizes the financial sector to coordinate on enforcement but also gives the platform market power to extract rents. In this sense, regulators have a "natural monopoly" dilemma. We close the paper by discussing how some classic policy responses would play out in our model: regulated competition between platforms and a public ledger option. We do this by varying the outside option with which the platform competes.

3.6.1 Regulated Platform Competition

Environment changes: The environment is the same as in subsection 3.1 but with the following changes. There are now two private platforms, labeled $n \in \{1, 2\}$.

There is no public marketplace or public currency. Both platforms manage their own ledger, charge a markup μ^n, and have average trading advantage ζ^n. For simplicity, we assume that the platforms choose a fixed μ at time $t = 0$ for all periods. We let $\eta_{\tau,t}$ denote the fraction of agents at age τ choosing platform 1.

Since there is no public dollar, agents cannot undertake side payments; all transactions are observed by one of the two platforms. In other words, in this new environment the only way producers can default is by writing a contract on the ledger provided by platform n, then defaulting and trading on the other platform n'. We also now assume that $1 - \chi$ captures the dead-weight loss producers incur when they default and lenders get nothing when default occurs. We use the currency provided by ledger 1 as the numeraire for asset pricing. So, ϵ_t now refers to the real exchange rate from tokens provided by platform 1 to tokens provided by platform 2.

Regulation: The regulator allows the platforms to bargain at time $t = 0$ over committing to exclude funds who allow their depositors to default on contracts on the other ledger. We assume that funds face no borrowing constraints or commitment problems during this bargaining and the the Nash bargaining protocol is followed. The regulator does not allow the platforms to collude on setting markups at times $t \geq 0$.

Platform competition at $t = 0$: For $t > 0$, the equilibrium is the same as in the subsection 3.3 but with $(1 - \mu_t)$ replaced by $(1 - \mu_t^1)/(1 - \mu_t^2)$. Let q_0^{En} denote the price of equity in platform n at $t = 0$ under cooperation on enforcement for $t \geq 0$ and let $\tilde{q}_0^{En}$ denote price of equity in platform n at $t = 0$ if there is no cooperation on enforcement for $t \geq 0$. The surplus from cooperation is $S = q_0^{E2} - \tilde{q}_0^{E2} + q_0^{E1} - \tilde{q}_0^{E1}$. If the surplus is positive, then the platforms bargain over coordination on contract enforcement. We assume that platform 1 makes a (positive or negative) transfer T to platform 2 at time 0 and the payment is determined by a Nash Bargaining protocol.

In particular, we have:

$$
\begin{aligned}
T = & \arg\max_T \left\{ \left(q_0^{E2} - T - \tilde{q}_0^{E2} \right) \left(q_0^{E1} + T - \tilde{q}_0^{E1} \right) \right\} \\
& = q_0^{E2} - \tilde{q}_0^{E2} - (q_0^{E1} - \tilde{q}_0^{E1})
\end{aligned}
$$

If the surplus is negative, then the platforms do not coordinate. Proposition 1 shows that, when platforms are symmetric, the outcome of the bargaining is contract enforcement on both ledgers whereas when platforms are asymmetric the dominant platform provides the ledger.

Proposition 1. *We have the following:*

(i) If the platforms are symmetric, then the outcome of the bargaining at time 0 *is that contracts are enforced on both ledgers and no transfer is made.*

(ii) If $\zeta := \zeta_\tau^1/\zeta_\tau^2 > 1$*, then for* χ *and* ζ *sufficiently high, the outcome of the bargaining at* $t = 0$ *is that platform 1 provides the monopoly ledger for the economy.*

Proof. See Appendix B. □

The first part of the proposition says that contract enforcement coordination is straightforward when the platforms are similar. The second part of theorem reinforces the market structure result in Section 2. Ultimately, it shows that the only ledger operators that are viable are those that also possess a platform trading technology. In other words, there is a natural bundling between offering ledger and trading services. This implies that a financial intermediary with no trading technology (which would be modeled as $\zeta_2 = 0$ in our environment) would never provide the ledger in equilibrium.

Given the outcome of the bargaining over contract enforcement, we show the numerical solution to symmetric platform competition for $t > 0$ in Figure 3. Evidently, we recover the perfectly competitive market outcome as $\gamma_1 \to \infty$ and the agents become highly elastic. In this sense, seller elasticity governs the extent to which we recover Bertrand style competition. This suggests two major lessons for financial competition regulators:

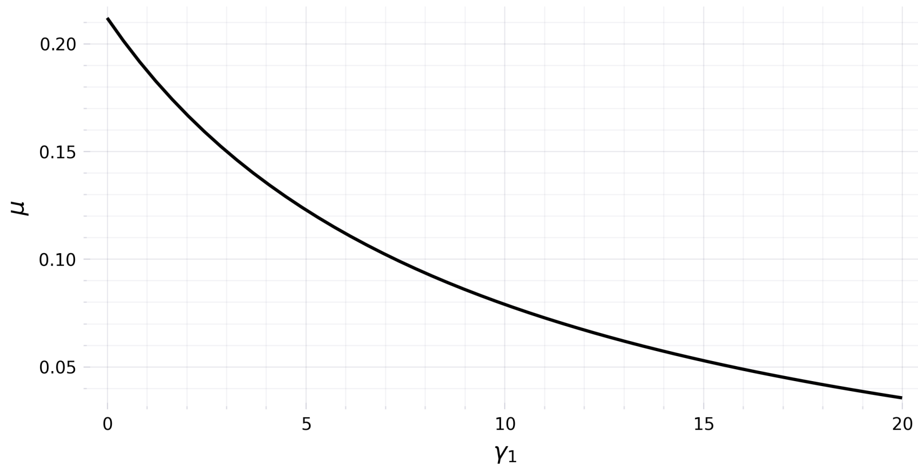

Figure 3: Ledger choice for $\gamma_1 \in [0, 20]$.

Other variables are $\gamma_2 = 0.5$, $z = 1.0$, $\alpha = 0.75$, $\rho = 1.0$, $\lambda = 1.0$, $\zeta = 1$, $\kappa = 0.1$, and $\chi = 0.5$.

(i) They should focus on understanding the elasticity with which buyers and sellers are able substitute between trading technologies.

(ii) Small platforms need to be able to commit to making payments to large platforms in order to get cooperation on an enforcement equilibrium.

3.6.2 Public Ledger Option

Environment changes: We now consider the environment from subsection 3.1 but with public money replaced by a digital ledger provided by the government that can be used by all intermediaries and agents in the economy. This can be interpreted as giving common access to central bank reserves, introducing a public CBDC, or allowing tech platforms access the FedNow payment system. The government commits to using payment revenue to enforce contracts written on the ledger. This means that there is no longer a cash-in-advance constraint on the public marketplace and contracts are enforced on the public marketplace if agents use the public ledger.

The platform now has additional relevant strategic options. As before, it could help ensure contract enforcement by forcing traders to pay using the public settlement ledger or their own ledger. However, it could also choose to provide a private token that replaces public money for hidden trades.

Corollary 2. *Let (R, Y) denote the steady state equilibrium interest rate and aggregate output from 3.5 when the government provides money and the platform provides the settlement ledger. (R^*, Y^*) denote the steady state equilibrium interest rate and aggregate output when the government provides a common digital ledger and the platform optimizes markups.*

(i) If the government creates a forced tender ledger that must be used for all payment, then all contracts are enforced, the cash-in-advance constraint is eliminated, $R^* < R$, and $Y^* > Y$.

(ii) If the government allows the platform to choose any payment technology, then the platform response depends upon their trading advantage. For sufficiently large ζ, the platform forces agents to use public ledger. For sufficiently small ζ, the platform prefers to offer its own private token that is only used for trade on the ledger and allow agents to default.

If it feasible for the government to create a forced tender ledger, then the economy ends up back in the world of Kocherlakota (1998). All agents are using a ledger run by a benevolent central planner ensuring that all contracts are enforced.

However, if the platform provides public ledger as a voluntary payment technology, then the response of the platform is more complicated. In our original model, the platform is offering the only payment technology where contracts can be enforced whereas the government is offering the payment technology that can be used for side-trading to avoid contract enforcement. We showed that the platform is strongly incentivized to ensure contract enforcement because otherwise there is no lending and production in the economy. When the government introduces a public ledger, this reverses. The government is now offering a ledger where contracts are always enforced and platform is deciding whether to offer the side-trading option where contracts are not enforced. That is, the platform is now deciding whether or not set up a "black-market" with a private token. The platform faces the trade-off that allowing default increase the interest rate in the economy thereby decreasing output but allowing default also attracts more producers to trade on the platform. What the platform decides depends upon their market power.

In our environment, introducing a public ledger option is potentially powerful but also offers a few points of caution for regulators:

(i) A forced tender ledger is very powerful but raises privacy concerns.

(ii) Introducing a non-compulsory public ledger improves contract enforcement in the public marketplace but incentivizes the private platform to set up a "black-market" with its own private tokens where agents can default.

4 Conclusion

In this paper, we model the strategic decision making of a private controller of the currency ledger used for settling transactions and writing contracts. We find that in an unregulated economy "BigTech" platforms are likely to provide "FinTech" services. This brings both benefits and costs. Tech platforms can expand uncollateralized credit across a supply chain by exploiting their control of the payment system to better coordinate the financial system to enforce contracts. However, Tech platforms will also use their control of the ledger to increase their market power and charge high markups. We see these issues playing out in China where tech platforms Alibaba and WeChat have created a well-functioning payment system with very limited competition.

Ultimately, our model suggests that currency ledgers may need to be regulated like other natural monopolies. This could include restrictions on when ledgers can cooperate on contract enforcement and compete on markups. It could also include a competing public option in the form a programmable Central Bank Digital Currency (CBDC) ledger. We consider further modeling of the government's regulatory options as important future work.

References

Abadi, J. and Brunnermeier, M. (2018). Blockchain economics. Technical report, National Bureau of Economic Research.

Ahnert, T., Hoffmann, P., and Monet, C. (2022). The digital economy, privacy, and CBDC.

Aiyagari, S. R. and Wallace, N. (1991). Existence of steady states with positive consumption in the Kiyotaki-Wright model. *The Review of Economic Studies*, 58(5):901–916.

Benigno, P., Schilling, L. M., and Uhlig, H. (2019). Cryptocurrencies, currency competition, and the impossible trinity. Technical report, National Bureau of Economic Research.

Bigio, S. (2023). A theory of payments-chain crises. Technical report, National Bureau of Economic Research.

Brunnermeier, M. and Payne, J. (2023). Platform, ledgers, and interoperability. *Princeton Working Paper*.

Brunnermeier, M. K. and Sannikov, Y. (2019). International monetary theory: A risk perspective. Technical report.

Chiu, J. and Wong, R. (2020). Payments on digital platforms: Resilience, interoperability and welfare.

Cong, L. W., Li, Y., and Wang, N. (2020). Token-based platform finance. *Fisher College of Business Working Paper*, (2019-03):028.

Cong, L. W., Li, Y., and Wang, N. (2021). Tokenomics: Dynamic adoption and valuation. *The Review of Financial Studies*, 34(3):1105–1155.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.

Farhi, E., Golosov, M., and Tsyvinski, A. (2009). A theory of liquidity and regulation of financial intermediation. *The Review of Economic Studies*, 76(3):973–992.

Fernández-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2020). Central bank digital currency: Central banking for all? Technical report, National Bureau of Economic Research.

Fernández-Villaverde, J. and Sanches, D. R. (2018). On the economics of digital currencies.

Freeman, S. (1996a). Clearinghouse banks and banknote overissue. *Journal of Monetary Economics*, pages 101–115.

Freeman, S. (1996b). The payments system, liquidity, and rediscounting. *The American Economic Review*, pages 1126–1138.

Garber, P., Green, B., and Wolfram, C. (2021). Digital collateral. (28724).

Hayek, F. A. (1976). *Choice in Currency: A Way to Stop Inflation*, volume 48. Ludwig von Mises Institute.

Jacklin, C. J. (1987). Demand deposits, trading restrictions, and risk sharing.

Kahn, C. and van Oordt, M. R. (2022). The demand for programmable payments. Technical report, http://dx.doi.org/10.2139/ssrn.4218966.

Kahn, C. M., Rivadeneyra, F., and Wong, T.-N. (2019). Should the central bank issue e-money? *Money*, pages 01–18.

Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates. *The Quarterly Journal of Economics*, 96(2):207–222.

Keister, T. and Sanches, D. R. (2019). Should central banks issue digital currency?

Kocherlakota, N. R. (1998). Money is memory. *journal of economic theory*, 81(2):232–251.

Lagos, R., Rocheteau, G., and Wright, R. (2017). Liquidity: A new monetarist perspective. *Journal of Economic Literature*, 55(2):371–440.

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of political Economy*, 113(3):463–484.

Lucas Jr, R. E. and Stokey, N. L. (1985). Money and interest in a cash-in-advance economy. Technical report, National Bureau of Economic Research.

Rishabh, K. and Schäublin, J. (2021). Payment fintechs and debt enforcement.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy*, 66(6):467–482.

Schilling, L. and Uhlig, H. (2019). Some simple bitcoin economics. *Journal of Monetary Economics*, 106:16–26.

Svensson, L. E. (1985). Currency prices, terms of trade, and interest rates: A general equilibrium asset-pricing cash-in-advance approach. *Journal of International Economics*, 18(1-2):17–41.

A Supplementary Proofs to Section 2

Proof of Theorem 1. Suppose that the ledger is set up to enforce contracts and exclude agents who default but the platform allows agents to chose the use of payment technology ($\iota_s = 1$) and does not exclude agents from the trading technology ($\iota_d = 1$). Then at $t = 1$ and $t = 2$ all agents default.

Suppose that the platform prevents the use of spot trade ($\iota_s = 0$) and threatens to exclude defaulting agents but the ledger operator does not allow the ledger to used on the platform and does not share ledger information with the platform. Then, at $t = 1$ no agents can trade on the ledger and no trade occurs and at $t = 2$ the platform cannot identify the defaulting agents and so all or no agents are allowed to trade. In this case, we effectively have $\iota_p = 1$ and so all agents default. □

Proof of Theorem 2. In all cases the platform sets $\mu_1 = 1$ since there is no cost to extracting markups for trades at $t = 1$. For other choices, we have to consider a collection of four cases.

(1) First, suppose that the platform does not exclude spot trade or defaulters $\iota_s = \iota_p = 1$. Then all agents default, no loans are made, and the platform gets zero profit.

(2) Now, suppose that the platform excludes spot trade and defaulters $\iota_s = \iota_p = 0$. In this case, the IC constraint becomes:

$$\mu \leq 1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\psi z}{z-R} - 1\right)$$

There are two subcases. (Subcase a) If the platform wants contract enforcement, then it will choose the maximum μ that satisfies the constraint and so the total profit for the platform under contract enforcement is:

$$\eta\left((1-\beta)(z-R) + \bar{\mu}\beta(z-R)\right) \tag{A.1}$$

where $\bar{\mu} = \left(1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\zeta z}{z-R} - 1\right)\right)$, $R = 1/\phi$, and $\phi = (1-\beta)\eta + \beta$. This is feasible so long as $1/((1-\beta)\eta + \beta) \leq z$. (b) If the platform does not want contract enforcement, then it chooses $\mu_1 = 1$ and so profit is:

$$\eta\left((1-\beta)(z-\hat{R}) + \beta\psi z\right)$$

where $\hat{R} = 1/\hat{\phi}$, and $\hat{\phi} = (1-\beta)\eta$. This is feasible so long as $1/((1-\beta)\eta) \leq z$. So, the platform prefers contract enforcement iff:

$$\begin{aligned}
-\eta(1-\beta)R + \bar{\mu}\beta(z-R) &\geq -\eta(1-\beta)\hat{R} + \beta\psi z \\
-\eta(1-\beta)R - \bar{\mu}R\beta + \beta R - \beta R + \bar{\mu}\beta z &\geq -1 + \beta\psi z \\
\Leftrightarrow -1 - (\bar{\mu}-1)\beta R + \bar{\mu}\beta z &\geq -1 + \beta\psi z \\
\Leftrightarrow (1-\bar{\mu})\beta R + \bar{\mu}\beta z &\geq \beta\psi z
\end{aligned}$$

(3) Now, suppose that the platform does exclude spot trade $\iota_s = 0$ but does not exclude defaulters $\iota_d = 1$. Then the platform sets $\mu_1 = \mu_2 = 1$ and takes profit:

$$\eta((1-\beta)(z-R) + \beta\psi z) = \eta(((1-\beta) + \beta\psi)z - 1$$

where we have used that $\phi = \eta(1-\beta)$ and $R = 1/\phi$.

(4) Finally consider the case where the platform does not exclude spot trade $\iota_s = 1$ but does exclude defaulters $\iota_d = 0$. If the platform wants contract enforcement, then it will choose the maximum μ that satisfies the constraint and so the total profit for the platform under contract enforcement is:

$$\eta\left((1-\beta)\psi z + \bar{\mu}\beta(z-R)\right)$$

where $\bar{\mu} = \left(1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\zeta z}{z-R} - 1\right)\right)$, $R = 1/\phi$, and $\phi = \beta$. If the platform does not want contract enforcement, then there is no trade. So, the platform prefers contract enforcement iff:

$$\eta\left((1-\beta)\psi z + \bar{\mu}\beta(z-R)\right) \geq 0$$

To complete the proof, we need to consider the limiting cases. Suppose that $\eta \to 1$. Then $\bar{\mu} \to 1$ and $\phi \to \eta(1-\beta) + \beta \to 1$. In this case, equation (A.1) becomes the largest profit amongst all the cases.

For the other limiting case that $\eta \to 0$, we have that $\phi \to 0$ and $R \to \infty$. This means that cases (2b) and (3) with default on $t = 2$ trades are infeasible. We also have that $\bar{\mu} \to -\infty$ for cases (2a) and (3) so the platform would have to earn negative profits in order to incentivize contract enforcement. Thus, the platform does not want to set up the ledger. □

B Supplementary Proofs for Section 3(Online Appendix)

B.1 Discrete Choice Problems

This section of the appendix contains working for the discrete choice problems. Since these are standard results, we provided limited detail.

Lemma 2. *Let $\{\zeta^n\}_{n\leq N}$ be a collection of independent draws from $Gu(\gamma,\mu)$, where $\mu=-\gamma\mathcal{E}$ and $\mathcal{E}$ represents the Euler–Mascheroni constant. Let $u(c)=\log(c)$. Then:*

$$\max_{n\leq N}\{\zeta^n+\varphi^n u(\pi^n)\}\sim Gu\left(\gamma,\mu+\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right)\right) \tag{B.1}$$

and so we have:

$$\mathbb{E}[\max_n\{\zeta^n+\varphi^n\log(\pi^n)\}]=\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right),$$
$$\mathbb{P}\left(n=argmax_{n'}\left\{\zeta^{n'}+\varphi^{n'}\log(\pi^{n'})\right\}\right)=\frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}(\pi^{n'})^{\varphi^{n'}/\gamma}}$$

Proof. Using the definition of the Gumbel distribution and the independence of the N draws, we have that:

$$\begin{aligned}\mathbb{P}(\max_n\{\zeta^n+\varphi^n u(\pi^n)\}\leq k) &= \prod_n\mathbb{P}(\zeta^n+\varphi^n u(\pi^n)\leq k)\\ &= \exp\left(\sum_n -e^{-(k-\mu)/\gamma}e^{\varphi^n u(\pi^n)/\gamma}\right)\\ &= \exp\left(-e^{-\left(k-\mu-\gamma\log\left(\sum_n e^{\varphi^n u(\pi^n)/\gamma}\right)\right)/\gamma}\right)\end{aligned}$$

which implies result (B.1). From the properties of the Gumbel distribution, the expectation is:

$$\begin{aligned}\mathbb{P}\left(n=\operatorname{argmax}_{n'}\left\{\zeta^{n'}+\varphi^{n'}\log(\pi^{n'})\right\}\right) &= \left[\mu+\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right)\right]+\gamma\mathcal{E}\\ &=\gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right)\end{aligned}$$

and the probability of choosing n is:

$$\begin{aligned}\mathbb{P}(n = \operatorname{argmax}\{\zeta^{ni} + \varphi^n \log(\pi^n)\}) &= \frac{e^{\varphi^n u(\pi^n)/\gamma}}{\sum_{n'} e^{\varphi^{n'} u(\pi^{n'})/\gamma}} \\ &= \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'} (\pi^{n'})^{\varphi^{n'}/\gamma}}\end{aligned}$$

□

B.2 Other Proofs

Proof of Theorem 3. We solve the problem recursively. At age 2, taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (B.9) below (dropping the explicit i superscript and the time subscript on the choice n_2):

$$\begin{aligned}V_{2,t+2}(d) = \mathbb{E}\left[\max_{c,n}\left\{\zeta^n_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad c \leq R^n_{t+1,t+2} d/\epsilon^n_t, \quad \forall \mathcal{L} \in \{0,1\},\end{aligned} \tag{B.2}$$

where $V_{2,t+2}$ is the value function at the start of the agent's final period. Using standard discrete choice results (summarized in Lemma 2 in the Appendix), the value function satisfies:

$$V_{2,t+2}(d) = \log\left(\bar{\nu}_{2,t+2} d\right) \tag{B.3}$$

where the average purchasing power at time τ is:

$$\bar{\nu}_{2,t+2} := \left(\sum_{n'}\left(\zeta^{n'}_{t+2} R^{n'}_{t+1,t+2}/\epsilon^{n'}_{t+2}\right)^{\gamma^b}\right)^{1/\gamma^b}$$

and the fraction of buyers who choose n at time $t+2$ is given by:

$$\eta^n_{2,t+2} = \frac{\left(\zeta^n R^n_{t+1,t+2}/\epsilon^n_{t+2}\right)^{\gamma_2}}{\sum_{n'}\left(\zeta^{n'} R^{n'}_{t+1,t+2}/\epsilon^{n'}_{t+2}\right)^{\gamma_2}}$$

At age 1, after selling production goods, taking price processes as given, an agent who has made profit π in token goods selling on platform n solves the problem:

$$V_{1,t+1}^{n}(\pi) = \max_{c,d} \{(1-\beta)u(c) + \beta V_{2,t+2}(d)\}$$
$$s.t. \quad d \leq \pi - \epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})c,$$

Substituting in the constraint gives the standard consumption saving decision:

$$\max_{d} \left\{(1-\beta)u\left(\frac{\pi - d}{\epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})}\right) + \beta V_{2,t+2}(d)\right\}$$

The first order condition gives:

$$0 = -\frac{(1-\beta)u'(c_{1,t+1})}{\epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})} + \beta V_{2,t+2}'(d)$$

Imposing the functional forms and rearranging gives:

$$\frac{1-\beta}{\epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})c} = \frac{\beta}{d} = \frac{\beta}{\pi - \epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})c}$$

and so:

$$c_{1,t+2} = \frac{(1-\beta)\pi}{\epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})}, \qquad d_{1,t+1} = \beta\pi$$

And so we have:

$$\begin{aligned} V_{1,t+1}^{n}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon_{t+1}^{n}(1-\mu_{t+1}^{n})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+2}^{n}(1-\mu_{t+1}^{n})\right) + \log(\pi) \end{aligned}$$

Now consider the problem of an agent at age 0 during the morning market. They choose where to purchase input goods, where to sell output goods, and the quantity of input goods to solve (B.10) below:

$$V_{0,t} = \mathbb{E}_t\left[\max_{n_0,n_1,x,\pi}\left\{\zeta_{0,t}^{n_0} + \zeta_{1,t+1}^{n_1} + V_{1,t+1}^{n_1}(\pi)\right\}\right] \quad s.t. \tag{B.4}$$
$$\pi = \epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1})zx^{\alpha} - R_{t,t+1}^{bn_0}x$$

For a given choice of $\underline{\mathbf{n}}_1$, the agent choose x to maximize:

$$\max_x \left\{\epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1})zx^\alpha - R_{t,t+1}^{bn_0}x\right\}$$

Taking the FOC gives that producer labor demand, output, and profit are given by:

$$x_{0,t} = \left(\frac{\alpha z(1-\mu_{t+1}^{n_0})}{R_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad y_{1,t+1}^{n_0} = z\left(\frac{\alpha z(1-\mu_{t+1}^{n_1})}{R_{t,t+1}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t+1} = \left(\frac{\epsilon_{t+1}^{n_1}\alpha z(1-\mu_{t+1}^{n_1})}{(R_{t,t+1}^{bn_0})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

Returning to the value functions, we have that:

$$\begin{aligned}
V_{1,t+1}^n(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon_{t+1}^n(1-\mu_{t+1}^n)}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\
&= \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon_{t+2}^n(1-\mu_{t+1}^n)\right) + \log(\pi)
\end{aligned}$$

And so we have:

$$\begin{aligned}
V_{0,t} &= \mathbb{E}_t\left[\max_{n_0,n_1,x,\pi}\left\{\zeta_{0,t}^{n_0} + \zeta_{1,t+1}^{n_1} + V_{1,t+1}^{n_1}(\pi)\right\}\right] \\
&= \mathbb{E}_t\left[\max_{n_0,n_1}\left\{\zeta_{0,t}^{n_0} + \zeta_{1,t+1}^{n_1} - \log\left(\left(\epsilon_{t+2}^{n_1}(1-\mu_{t+1}^{n_1})\right)^{1-\beta}\right)\right.\right. \\
&\quad \left.\left. + \log\left(\left(\frac{\epsilon_{t+1}^{n_1}\alpha z(1-\mu_{t+1}^{n_1})}{(R_{t,t+1}^{bn_0})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)\right\}\right] \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) \\
&= \mathbb{E}_t\left[\max_{n_0,n_1}\left\{\zeta_{0,t}^{n_0} + \zeta_{1,t+1}^{n_1} + \log\left(\frac{\left(\epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{\mathcal{L}'})\right)^{\frac{1}{1-\alpha}-(1-\beta)}}{\left(R_{t,t+1}^{bn_0}\right)^{\frac{\alpha}{1-\alpha}}}\right)\right\}\right] \\
&\quad + \log\left((\alpha z)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right) \\
&\quad + \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right)
\end{aligned}$$

and Lemma 2 implies that the fraction of agents at age 0 choosing to purchase inputs on n_0 and at age 1 choosing to purchase on n_1 satisfies:

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}(R_{t,t+1}^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}(R_{t,t+1}^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}$$

$$\eta_{1,t+1}^{n_1} = \frac{\left(\zeta_1^{n_1}\left(\epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_{t+1}^{n_1'}(1-\mu_{t+1}^{n_1'})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}}$$

and:

$$\begin{aligned} V_{0,t} = {} & \log(\bar{\nu}_{0,t}) + \log(\bar{\nu}_{1,t+1}) + \beta\log\left(\bar{\nu}_{2,t+2}\right) \\ & + \log\left((\alpha z)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right) + \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) \end{aligned}$$

where:

$$\bar{\nu}_{0,t} := \left(\sum_{n_0}\left(\zeta_{0,t}^{n_0}\left(R_{t,t+1}^{bn_0}\right)^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}\right)^{1/\gamma_0}$$

$$\bar{\nu}_{1,t+1} := \left(\sum_{n_1}\left(\zeta^{n_1}\left(\epsilon_{t+1}^{n_1}(1-\mu_{t+1}^{n_1})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_2}\right)^{1/\gamma_2}$$

□

Proof of Lemma 1. We start by setting up the constraints on the fund. The return for depositors withdrawing to trade on the public marketplace is:

$$R_{t,t+1}^{do} = \frac{q_{t+1}^m}{q_t^m}. \tag{B.5}$$

and so the "money-in-advance" constraint on the fund is:

$$\eta_{2,t+1}^m R_{t,t+1}^{do} D_t \leq q_{t+1}^m M_t \quad \Leftrightarrow \quad \eta_{2,t+1}^o D_t \leq q_t^m M_t \tag{B.6}$$

Now consider the return for the other depositors. Let $A_t := q_t^m(M_t - \eta_{2,t+1}^o D_t/q_t^m) + q_t^b B_t + q_t^s S_t$. After money withdrawals, the remaining wealth is:

$$A_{t+1} = (q_{t+1}^b \mathcal{E}_{t+1})(M_t - \eta_{2,t+1}^m D_t/q_t^m) + B_t + (q_{t+1}^s + \pi_{t+1}^s)S_t$$

and so the return for depositors trading on the platform is:

$$R^{dp}_{t,t+1} = \frac{A_{t+1}}{A_t} = \frac{q^b_{t+1}\mathcal{E}_{t+1}}{q^m_t}\theta^M_t + \frac{1}{q^b_t}\theta^B_t + \frac{q^s_{t+1} + \pi^s_{t+1}}{q^s_t}\theta^{B,1}_t \tag{B.7}$$

where $\theta^M_t := q^m_t(M_t - \eta^m_{2,t+1}D_t/q^m_t)/A_t$, $\theta^B_t := q^b_t B_t/A_t$, and $\theta^S_t := q^s_t S_t/A_t$.

The fund maximizes its return to depositors (B.8) below:

$$\begin{aligned} &\max_{M_t,B_t,S_t} \left\{\xi_{t,t+1}(\eta^o_{2,t+1}R^{dp}_{t,t+1} + \eta^p_{2,t+1}R^{dp}_{t,t+1})D_t\right\} \\ &\quad s.t. \quad (3.9), (\text{B.5}), (\text{B.6}), (\text{B.7}) \end{aligned} \tag{B.8}$$

where $\xi_{t,t+1} := \left(\frac{1-\beta}{\beta}\right) V_{2,t+2}(a)/\partial_c u'(c_{2,t+2})$ is the agent discount factor. Thus, in equilibrium, we must have that the return on holding money to make dollar loans in the morning market, bonds, and equity are the same:

$$\frac{q^b_{t+1}\mathcal{E}_{t+1}}{q^m_t} = \frac{1}{q^b_t} = \frac{q^s_{t+1} + \pi^s_{t+1}}{q^s_t}$$

In particular, this implies that the price of loans in m-goods is:

$$\mathcal{E}_{t+1} = \frac{q^m_t}{q^b_{t+1}}\frac{1}{q^b_t} = E_{t+1}\frac{E_t q^b_t}{E_{t+1}q^b_{t+1}}\frac{1}{q^b_t} = E_{t+1}\frac{R^b_{t,t+1}}{R^m_{t,t+1}}$$

and so:

$$q^{bm}_t = \frac{1}{\mathcal{E}_t P^m_t} = \frac{1}{E_t P^m_t}\frac{E_t}{\mathcal{E}_t} = \frac{q^b_t}{\epsilon_t}\frac{E_t}{\mathcal{E}_t} = \frac{q^b_t}{\epsilon_t}\frac{R^m_{t-1,t}}{R^b_{t-1,t}}$$

which implies that:

$$R^{bm}_{t,t+1} = \epsilon_t R^b_t \frac{R^b_{t-1,t}}{R^m_{t-1,t}}$$

□

Proof of Theorem 4. Summarising the equilibrium and optimization equations gives the following characterization of equilibrium. Given states $\{M_t, \mu_{t-2}, \mu_{t-1}, R_{t-1}\}$ and current policy μ_t, we can solve for the equilibrium variables at time t:

$$\left(\underline{\mathbf{c}}_{1,t}^{\mathbf{n}_t}, \underline{\mathbf{c}}_{2,t}^{\mathbf{n}_t}, \underline{\mathbf{x}}_t^{\mathbf{n}_{t+1}}, \underline{\mathbf{y}}_t^{\mathbf{n}_t}, \underline{\boldsymbol{\pi}}_t^{\mathbf{n}_t}, \underline{\mathbf{d}}_t^{\mathbf{n}_t}, \epsilon_t, R_t^{bn}, R_t^{dn}, \underline{\boldsymbol{\eta}}_t^{\mathbf{n}_t}\right)$$

using the equations for agent choices:

$$x_{0,t}^{\mathbf{n}_1} = \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{R_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad y_{1,t}^{\mathbf{n}_1} = z\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{R_{t-1,t}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t}^{\mathbf{n}_1} = \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{(R_{t-1,t}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \qquad c_{1,t}^{\mathbf{n}_1} = \frac{(1-\beta)\pi_{1,t}}{(1-\mu_t^{n_1})\epsilon_t^{n_1}}$$

$$d_{1,t}^{\mathbf{n}_1} = \beta\pi_{1,t}^{\mathbf{n}_1}, \qquad c_{2,t}^{\mathbf{n}_2} = \frac{R_{t-1,t}^{dn_2}\beta\pi_{1,t-1}^{\mathbf{n}_1}}{\epsilon_t^{n_2}}.$$

$$\eta_{0,t}^{n_0} = \frac{\left(\zeta_0^{n_0}(R_t^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta_0^{n_0'}(R_t^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}, \qquad \eta_{2,t}^{n_2} = \frac{\left(\zeta_2^{n_2}R_{t-1,t}^{dn_2}/\epsilon_t^{n_2}\right)^{\gamma_2}}{\sum_{\mathcal{L}'}\left(\zeta_2^{n_2'}R_{t-1,t}^{dn_2'}/\epsilon_t^{n_2'}\right)^{\gamma_2}}$$

$$\eta_{1,t}^{n_1} = \frac{\left(\zeta_1^{n_1}\left(\epsilon_t^{n_1}(1-\mu_t^{n_1})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta_1^{n_1'}\left(\epsilon_t^{n_1'}(1-\mu_t^{n_1'})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}},$$

fund equations:

$$R_{t,t+1}^{bn} = \epsilon_t^n\frac{R_{t-1,t}^b}{R_{t-1,t}^n}R_{t,t+1}^b \qquad R_{t,t+1}^{dn} = R_{t,t+1}^n$$

$$q_t^{\$}M_t + q_t^bB_t + q_t^sS_t = D_t \qquad \eta_{2,t+1}^m D_t = q_t^{\$}M_t$$

and the market clearing conditions:

$$\begin{aligned}
\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}y^{(n_0,n)}_{1,t} &= \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}c^{(n_0,n)}_{1,t} \\
&\quad + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t} \\
D_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}d^{(n_0,n_1)}_{1,t} \\
q^b_t B_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t x^{(n_0,n_1)}_{0,t} \\
q^\$_t M_t &= \eta^m_{2,t}D_t + \sum_{n_1}\eta^m_{0,t}\frac{q^b_t}{q^{bm}_t}x^{(m,n_1)}_{0,t} \\
S_t &= 1
\end{aligned}$$

<u>(i) Solve for ϵ_t</u>: We start by solving for ϵ_t. Substituting the fund returns into the agent choices gives:

$$\eta^{n_0}_{0,t} = \frac{\left(\zeta^{n_0}_0(R^{n_0}_{t-1,t}/\epsilon^{n_0}_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n'_0}\left(\zeta^{n'_0}_0(\epsilon^{n'_0}_t/R^{n'_0}_{t-1,t})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \qquad \eta^{n_2}_{2,t} = \frac{\left(\zeta^{n_2}_2 R^{n_2}_{t-1,t}/\epsilon^{n_2}_t\right)^{\gamma_2}}{\sum_{\mathcal{L}'}\left(\zeta^{n'_2}_2 R^{n'_2}_{t-1,t}/\epsilon^{n'_2}_t\right)^{\gamma_2}}$$

Now, return to the goods market clearing condition. After rearranging, we have:

$$\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}\left(y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t}\right) = \eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t} + \eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}$$

where the LHS can be computed using:

$$\begin{aligned}
&y^{(n_0,n)}_{1,t} - c^{(n_0,n)}_{1,t} \\
&\quad = z\left(\frac{\alpha z(1-\mu^n_t)\epsilon^n_t}{R^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} - \frac{(1-\beta)}{(1-\mu^n_t)\epsilon^n_t}\left(\frac{\alpha z(1-\mu^n_t)\epsilon^n_t}{(R^{bn_0}_{t-1,t})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \\
&\quad = (1-(1-\beta)(1-\alpha))\,z\left(\frac{\alpha z(1-\mu^n_t)\epsilon^n_t}{R^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \\
&\quad = (1-(1-\beta)(1-\alpha))\,z\left(\frac{\alpha z(1-\mu^n_t)\epsilon^n_t}{\epsilon^{n_0}_{t-1}\frac{R^b_{t-2,t-1}}{R^{n_0}_{t-2,t-1}}R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \\
&\quad = (1-(1-\beta)(1-\alpha))\,y^{(n_0,n)}_{1,t}
\end{aligned}$$

and so:

$$\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}\left(y^{(n_0,n)}_{1,t}-c^{(n_0,n)}_{1,t}\right)$$

$$=\eta^n_{1,t}\sum_{n_0}\eta^{n_0}_{0,t-1}\left(1-(1-\beta)(1-\alpha)\right)z\left(\frac{\alpha z(1-\mu^n_t)\epsilon^n_t}{R^{bn_0}_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$=\eta^n_{1,t}\left(1-(1-\beta)(1-\alpha)\right)z(\alpha z(1-\mu^n_t)\epsilon^n_t)^{\frac{\alpha}{1-\alpha}}\sum_{n_0}\frac{\eta^{n_0}_{0,t-1}}{R^{bn_0}_{t-1,t}}$$

and where the RHS can be computed using:

$$\eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}x^{(n,n_1)}_{0,t}+\eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}c^{(n_0,n_1,n)}_{2,t}$$

$$=\eta^n_{0,t}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z(1-\mu^{n_1}_{t+1})\epsilon^{n_1}_{t+1}}{R^{bn}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}$$

$$+\eta^n_{2,t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\frac{R^{dn}_{t-1,t}\beta}{\epsilon^n_t}\left(\frac{\alpha z(1-\mu^{n_1}_{t-1})\epsilon^{n_1}_{t-1}}{(R^{bn_0}_{t-2,t-1})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

$$=\frac{\left(\zeta^n_0(R^n_{t-1,t}/\epsilon^n_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n'}\left(\zeta^{n'}_0(R^{n'}_{t-1,t}/\epsilon^{n'}_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z R^n_{t-1,t}(1-\mu^{n_1}_{t+1})\epsilon^{n_1}_{t+1}}{\epsilon^n_t R^b_{t-1,t}R^b_{t,t+1}}\right)^{\frac{1}{1-\alpha}}$$

$$+\frac{\left(\zeta^n_2 R^n_{t-1,t}/\epsilon^n_t\right)^{\gamma_2}}{\sum_{n'}\left(\zeta^{n'}_2 R^{n'}_{t-1,t}/\epsilon^{n'}_t\right)^{\gamma_2}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\frac{R^n_{t-1,t}\beta}{\epsilon^n_t}\left(\frac{\alpha z(1-\mu^{n_1}_{t-1})\epsilon^{n_1}_{t-1}}{(R^{bn_0}_{t-2,t-1})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

$$=\frac{\left(\zeta^n_0(R^n_{t-1,t}/\epsilon^n_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n'}\left(\zeta^{n'}_0(R^{n'}_{t-1,t}/\epsilon^{n'}_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}\left(\frac{R^n_{t-1,t}}{\epsilon^n_t}\right)^{\frac{1}{1-\alpha}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z(1-\mu^{n_1}_{t+1})\epsilon^{n_1}_{t+1}}{R^b_{t-1,t}R^b_{t,t+1}}\right)^{\frac{1}{1-\alpha}}$$

$$+\frac{\left(\zeta^n_2 R^n_{t-1,t}/\epsilon^n_t\right)^{\gamma_2}}{\sum_{n'}\left(\zeta^{n'}_2 R^{n'}_{t-1,t}/\epsilon^{n'}_t\right)^{\gamma_2}}\frac{R^n_{t-1,t}\beta}{\epsilon^n_t}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\left(\frac{\alpha z(1-\mu^{n_1}_{t-1})\epsilon^{n_1}_{t-1}}{(R^{bn_0}_{t-2,t-1})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

$$=(\zeta^n_0)^{\gamma_0}\left(\frac{R^n_{t-1,t}}{\epsilon^n_t}\right)^{\frac{1+\alpha\gamma_0}{1-\alpha}}\frac{1}{\bar{\nu}^{\gamma_0}_{0,t}}\sum_{n_1}\eta^{n_1}_{1,t+1}\left(\frac{\alpha z(1-\mu^{n_1}_{t+1})\epsilon^{n_1}_{t+1}}{R^b_{t-1,t}R^b_{t,t+1}}\right)^{\frac{1}{1-\alpha}}$$

$$+(\zeta^n_2)^{\gamma_2}\left(\frac{R^n_{t-1,t}}{\epsilon^n_t}\right)^{1+\gamma_2}\frac{1}{\bar{\nu}^{\gamma_2}_{2,t}}\sum_{n_0,n_1}\eta^{n_0}_{0,t-2}\eta^{n_1}_{1,t-1}\left(\frac{\alpha z(1-\mu^{n_1}_{t-1})\epsilon^{n_1}_{t-1}}{(R^{bn_0}_{t-2,t-1})^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

Under the assumption that $\frac{1+\alpha\gamma_0}{1-\alpha}=1+\gamma_2$ and $(\zeta^n_0)^{\gamma_0}=(\zeta^n_2)^{\gamma_2}$, we can take out the

n specific component to get:

$$\eta_{0,t}^{n} \sum_{n_1} \eta_{1,t+1}^{n_1} x_{0,t}^{(n,n_1)} + \eta_{2,t}^{n} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} c_{2,t}^{(n_0,n_1,n)}$$
$$= (\zeta_2^n)^{\gamma_2} \left(\frac{R_{t-1,t}^n}{\epsilon_t^n}\right)^{1+\gamma_2} \left[\frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\alpha z(1-\mu_{t+1}^{n_1})\epsilon_{t+1}^{n_1}}{R_{t-1,t}^b R_{t,t+1}^b}\right)^{\frac{1}{1-\alpha}}\right.$$
$$\left. + \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \left(\frac{\alpha z(1-\mu_{t-1}^{n_1})\epsilon_{t-1}^{n_1}}{(R_{t-2,t-1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)\right]$$

So, the market clearing condition in goods market n becomes:

$$\eta_{1,t}^{n} \left(1-(1-\beta)(1-\alpha)\right) z(\alpha z(1-\mu_t^n)\epsilon_t^n)^{\frac{\alpha}{1-\alpha}} \sum_{n_0} \frac{\eta_{0,t-1}^{n_0}}{R_{t-1,t}^{bn_0}}$$
$$= (\zeta_2^n)^{\gamma_2} \left(\frac{R_{t-1,t}^n}{\epsilon_t^n}\right)^{1+\gamma_2} \left[\frac{1}{\bar{\nu}_{0,t}^{\gamma_0}} \sum_{n_1} \eta_{1,t+1}^{n_1} \left(\frac{\alpha z(1-\mu_{t+1}^{n_1})\epsilon_{t+1}^{n_1}}{R_{t-1,t}^b R_{t,t+1}^b}\right)^{\frac{1}{1-\alpha}}\right.$$
$$\left. + \frac{1}{\bar{\nu}_{2,t}^{\gamma_2}} \sum_{n_0,n_1} \eta_{0,t-2}^{n_0} \eta_{1,t-1}^{n_1} \left(\frac{\alpha z(1-\mu_{t-1}^{n_1})\epsilon_{t-1}^{n_1}}{(R_{t-2,t-1}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)\right]$$

Dividing the market clearing condition in market n by the market clearing condition in market n' gives:

$$\left(\frac{\zeta_1^n \left(\epsilon_t^n(1-\mu_t^n)\right)^{\frac{1}{1-\alpha}+\beta-1}}{\zeta_1^{n'} \left(\epsilon_t^{n'}(1-\mu_t^{n'})\right)^{\frac{1}{1-\alpha}+\beta-1}}\right)^{\gamma_1} \left(\frac{(1-\mu_t^n)\epsilon_t^n}{(1-\mu_t^{n'})\epsilon_t^{n'}}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\zeta_2^n}{\zeta_2^{n'}}\right)^{\gamma_2} \left(\frac{R_{t-1,t}^n/\epsilon^n}{R_{t-1,t}^{n'}/\epsilon^{n'}}\right)^{1+\gamma_2}$$

which implies that:

$$\left(\frac{\epsilon_t^n}{\epsilon_t^{n'}}\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)+1+\gamma_2} \left(\frac{1-\mu_t^n}{1-\mu_t^{n'}}\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)} = \left(\frac{\zeta_2^n}{\zeta_2^{n'}}\right)^{\gamma_2} \left(\frac{\zeta_1^n}{\zeta_1^{n'}}\right)^{-\gamma_1} \left(\frac{R_{t-1,t}^n}{R_{t-1,t}^{n'}}\right)^{1+\gamma_2}$$

which gives:

$$\frac{\epsilon_t^n}{\epsilon_t^{n'}} = \left[\left(\frac{\zeta_2^n}{\zeta_2^{n'}}\right)^{\gamma_2} \left(\frac{\zeta_1^n}{\zeta_1^{n'}}\right)^{-\gamma_1} \left(\frac{R_{t-1,t}^n}{R_{t-1,t}^{n'}}\right)^{1+\gamma_2} \left(\frac{1-\mu_t^{n'}}{1-\mu_t^n}\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}\right]^{\frac{1}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}}$$

and so:

$$\epsilon_t = \left[\zeta_1^{\gamma_1}\zeta_2^{-\gamma_2}\left(\frac{R^{\$}_{t-1,t}}{R^b_{t-1,t}}\right)^{1+\gamma_2}(1-\mu_t)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

Let $\hat{\zeta}^n := \zeta^n/\zeta^{n'}$, $\hat{R}^n_{t,t+1} := R^n_{t,t+1}/R^{n'}_{t,t+1}$, and $\hat{\epsilon}^n_t := \epsilon^n_t/\epsilon^{n'}_t$. So, that:

$$\hat{\epsilon}^n_t = \left[\left(\hat{\zeta}^n\right)^{\gamma_2-\gamma_1}\left(\hat{R}^n_{t-1,t}\right)^{1+\gamma_2}\left(\hat{\mu}^n_t\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

and

$$\frac{\hat{\epsilon}^n_t}{\hat{R}^n_{t-1,t}} = \left[\left(\hat{\zeta}^n\right)^{\gamma_2-\gamma_1}\left(\frac{\hat{\mu}^n_t}{\hat{R}^n_{t-1,t}}\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

Returning agent decisions, we have that;

$$\eta^{n_0}_{0,t} = \frac{\left(\zeta^{n_0}(R^{n_0}_{t-1,t}/\epsilon^{n_0}_t)^{\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta^{n_0'}(\epsilon^{n_0'}_t/R^{n_0'}_{t-1,t})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} = \frac{1}{1+\left(\left(\hat{\epsilon}^{n_0}_t/\hat{R}^{n_0}_{t-1,t}\right)^{\frac{1}{1-\alpha}}/\hat{\zeta}^n\right)^{\gamma_0}}$$

$$\eta^{n_2}_{2,t} = \frac{\left(\zeta^{n_2}R^{n_2}_{t-1,t}/\epsilon^{n_2}_t\right)^{\gamma_2}}{\sum_{\mathcal{L}'}\left(\zeta^{n_2'}R^{n_2'}_{t-1,t}/\epsilon^{n_2'}_t\right)^{\gamma_2}} = \frac{1}{1+\left(\left(\hat{\epsilon}^{n_2}_t/\hat{R}^{n_2}_{t-1,t}\right)/\hat{\zeta}^n\right)^{\gamma_2}}$$

(ii) Solve for $R^b_{t,t+1}$: We have the following equations for the fund budget constraint, the bond market, the money market, the deposit market, and the equity market:

$$\begin{aligned}
D_t &= q^b_t B_t + q^{\$}_t M_t + q^s_t S_t \\
q^b_t B_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t}\eta^{n_1}_{1,t+1}\epsilon^{n_0}_t x^{(n_0,n_1)}_{0,t} \\
q^{\$}_t M_t &= \eta^m_{2,t}D_t + \sum_{n_1}\eta^m_{0,t}\eta^{n_1}_{1,t+1}\epsilon_t\frac{R^b_{t-1,t}}{R^{\$}_{t-1,t}}x^{(m,n_1)}_{0,t} \\
D_t &= \sum_{n_0,n_1}\eta^{n_0}_{0,t-1}\eta^{n_1}_{1,t}d^{(n_0,n_1)}_{1,t} \\
S_t &= 1
\end{aligned}$$

Sub into the fund budget constraint:

$$\sum_{n_0,n_1} \eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)} = \sum_{n_0,n_1} \eta_{0,t}^{n_0}\eta_{1,t+1}^{n_1}\epsilon_t^{n_0} x_{0,t}^{(n_0,n_1)} + \eta_{2,t}^{m} \sum_{n_0,n_1} \eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)}$$
$$+ \sum_{n_1} \eta_{0,t}^{m}\eta_{1,t+1}^{n_1}\epsilon_t \frac{R_{t-1,t}^b}{R_{t-1,t}^m} x_{0,t}^{(m,n_1)} + q_t^s$$

and so after rearranging we have:

$$\begin{aligned}
&(1-\eta_{2,t}^m)\sum_{n_0,n_1} \eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1} d_{1,t}^{(n_0,n_1)} \\
&\quad = \sum_{n_1} \eta_{1,t+1}^{n_1}\left(\sum_{n_0} \eta_{0,t}^{n_0}\epsilon_t^{n_0} x_{0,t}^{(n_0,n_1)} + \eta_{0,t}^{m}\epsilon_t \frac{R_{t-1,t}^b}{R_{t-1,t}^m} x_{0,t}^{(m,n_1)}\right) + q_t^s \\
&\quad = \sum_{n_1} \eta_{1,t+1}^{n_1}\left(\eta_{0,t}^{m}\epsilon_t^{m}\left(1+\frac{R_{t-1,t}^b}{R_{t-1,t}^m}\right) x_{0,t}^{(m,n_1)} + \eta_{0,t}^{p}\epsilon_t^{p} x_{0,t}^{(p,n_1)}\right) + q_t^s
\end{aligned}$$

Substituting returns into agent decisions gives:

$$x_{0,t}^{\mathbf{n}_1} = \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^{n_0}\frac{R_{t-1,t}^b}{R_{t-1,t}^{n_0}} R_{t,t+1}^b}\right)^{\frac{1}{1-\alpha}} \qquad d_{1,t}^{\mathbf{n}_1} = \beta\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{(\epsilon_t^{n_0}\frac{R_{t-2,t}^b}{R_{t-2,t}^{n_0}} R_{t-1,t}^b)^{\alpha}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)$$

So, we have that:

$$(1-\eta_{2,t}^m)\sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}\beta\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{(\epsilon_t^{n_0}\frac{R_{t-2,t}^b}{R_{t-2,t}^{n_0}}R_{t-1,t}^b)^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

$$=\sum_{n_1}\eta_{1,t+1}^{n_1}\left(\eta_{0,t}^m\epsilon_t^m\left(1+\frac{R_{t-1,t}^b}{R_{t-1,t}^m}\right)\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^m\frac{R_{t-1,t}^b}{R_{t-1,t}^m}R_{t,t+1}^b}\right)^{\frac{1}{1-\alpha}}+\eta_{0,t}^p\epsilon_t^p\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^p\frac{R_{t-1,t}^b}{R_{t-1,t}^p}R_{t,t+1}^b}\right)^{\frac{1}{1-\alpha}}\right)+$$

$$\Rightarrow\left(R_{t,t+1}^b\right)(1-\eta_{2,t}^m)^{1-\alpha}\left(\sum_{n_0,n_1}\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}\beta\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\left(\epsilon_t^{n_0}\frac{R_{t-2,t}^b}{R_{t-2,t}^{n_0}}\right)^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\right)^{1-\alpha}$$

$$=\left(R_{t-1,t}^b\right)^\alpha\left(\sum_{n_1}\eta_{1,t+1}^{n_1}\left(\eta_{0,t}^m\epsilon_t^m\left(1+\frac{R_{t-1,t}^b}{R_{t-1,t}^m}\right)\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^m\frac{R_{t-1,t}^b}{R_{t-1,t}^m}}\right)^{\frac{1}{1-\alpha}}\right.\right.$$

$$\left.\left.+\eta_{0,t}^p\epsilon_t^p\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^p\frac{R_{t-1,t}^b}{R_{t-1,t}^p}}\right)^{\frac{1}{1-\alpha}}\right)+(R_{t,t+1}^b)^{\frac{1}{1-\alpha}}q_t^s\right)^{1-\alpha}$$

$$\Rightarrow\frac{R_{t,t+1}^b}{\left(R_{t-1,t}^b\right)^\alpha}=\left(\frac{\sum_{n_1}\eta_{1,t+1}^{n_1}\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\frac{R_{t-1,t}^b}{R_{t-1,t}^m}}\right)^{\frac{1}{1-\alpha}}\left(\eta_{0,t}^m\epsilon_t^m\left(1+\frac{R_{t-1,t}^b}{R_{t-1,t}^m}\right)\left(\frac{1}{\epsilon_t^m}\right)^{\frac{1}{1-\alpha}}+\eta_{0,t}^p\right)+(R_{t,t+1}^b)^{\frac{1}{1-\alpha}}q_t^s}{\eta_{2,t}^p\sum_{n_0,n_1}\beta\eta_{0,t-1}^{n_0}\eta_{1,t}^{n_1}\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\left(\epsilon_t^{n_0}\frac{R_{t-2,t}^b}{R_{t-2,t}^{n_0}}\right)^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)}\right)^{1-\alpha}$$

In the steady state, we have:

$$\overline{R}=\frac{\sum_{n_1}\overline{\eta}_1^{n_1}\left(\frac{\alpha z(1-\mu^{n_1})\overline{\epsilon}^{n_1}}{\frac{\overline{R}^b}{\overline{R}^m}}\right)^{\frac{1}{1-\alpha}}\left(\overline{\eta}_0^m\overline{\epsilon}^m\left(1+\frac{\overline{R}^b}{\overline{R}^m}\right)\left(\frac{1}{\overline{\epsilon}^m}\right)^{\frac{1}{1-\alpha}}+\overline{\eta}^p\left(\frac{1}{\overline{\epsilon}^p}\right)^{\frac{1}{1-\alpha}}\right)+\overline{R}^{\frac{1}{1-\alpha}}\overline{q}^s}{(1-\overline{\eta}_2^m)\sum_{n_0,n_1}\beta\overline{\eta}_0^{n_0}\overline{\eta}_1^{n_1}\left(\frac{\alpha z(1-\mu^{n_1})\overline{\epsilon}^{n_1}}{\left(\overline{\epsilon}^{n_0}\frac{\overline{R}^b}{\overline{R}^{n_0}}\right)^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)}$$

Return on Money: Now return to the money market:

$$
\begin{aligned}
q_t^m M_t &= \eta_{2,t}^m D_t + \sum_{n_1} \eta_{0,t}^m \eta_{1,t+1}^{n_1} \epsilon_t \frac{R_{t-1,t}^b}{R_{t-1,t}^m} x_{0,t}^{(m,n_1)} \\
&= \eta_{2,t}^m \sum_{n_0,n_1} \eta_{0,t-1}^{n_0} \eta_{1,t}^{n_1} \beta \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{(\epsilon_t^{n_0} \frac{R_{t-2,t}^b}{R_{t-2,t}^{n_0}} R_{t-1,t}^b)^\alpha} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \\
&\quad + \sum_{n_1} \eta_{0,t}^m \eta_{1,t+1}^{n_1} \epsilon_t \frac{R_{t-1,t}^b}{R_{t-1,t}^m} \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{\epsilon_t^m \frac{R_{t-1,t}^b}{R_{t-1,t}^m} R_{t,t+1}^b} \right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

and:

$$
R_{t,t+1}^m = \frac{q_{t+1}^m}{q_t^m} = \left(\frac{M_t}{M_{t+1}}\right) \left(\frac{\eta_{2,t+1}^m D_{t+1} + \sum_{n_1} \eta_{0,t+1}^m \eta_{1,t+2}^{n_1} \epsilon_{t+1} \frac{R_{t,t+1}^b}{R_{t,t+1}^m} x_{0,t+1}^{(m,n_1)}}{\eta_{2,t}^m D_t + \sum_{n_1} \eta_{0,t}^m \eta_{1,t+1}^{n_1} \epsilon_t \frac{R_{t-1,t}^b}{R_{t-1,t}^m} x_{0,t}^{(m,n_1)}} \right)
$$

so in the steady state, this is:

$$
\overline{R}^m = 1/\mu^M
$$

Equity Price: The equity price satisfies:

$$
q_t^s = \frac{1}{R_{t,t+1}} \left(\pi_{t+1}^s + q_{t+1}^s\right)
$$

In the steady state, this implies that:

$$
\bar{q}^s = \frac{\bar{\pi}^s}{\bar{R}^b - 1}
$$

$\square$

Proof of Theorem 5. We consider the value for an agent who chooses to default when other agents are choosing to not default. We solve the problem recursively. At age 2, an agent who has defaulted and joined a fund taking defaulting agents can only hold cash and so only trade on the public marketplace. Taking price processes as given, an agent with deposits d chooses on which platform to search to solve problem (B.9) below (dropping the explicit i superscript and the time subscript on the choice n_2):

$$\check{V}_{2,t+2}(d) = \mathbb{E}\left[\max_{c}\left\{\zeta^{o}_{2,t+2} + u(c)\right\}\right] \\ s.t. \quad c \leq R^{o}_{t+1,t+2}d/\epsilon^{o}_{t+2}, \tag{B.9}$$

where $V_{2,t+2}$ is the value function at the start of the agent's final period. Evaluating this expression gives:

$$\check{V}_{2,t+2}(d) = \log\left(\check{\nu}_{2,t+2}d\right)$$

where $\check{\nu}_{2,t+2} := R_{t+1,t+2}/\epsilon_{t+2}$.

At age 1, the agent cannot default if they end up trading on the private platform. So, their value is:

$$V^{p}_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta V_{2,t+2}\left(d\right)\right\} \\ s.t. \quad d \leq \pi - (1-\mu_{t+1})c,$$

and so we have:

$$\begin{aligned} V^{p}_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^{p}_{t+1}(1-\mu^{p}_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^{p}_{t+2}(1-\mu^{p}_{t+1})\right) + \log(\pi) \end{aligned}$$

However, if they trade on the public marketplace, then they can choose cash trades, default, and go to a bank accepting cash trades without reporting them to the ledger. In this case, their value at $t = 1$ is given by:

$$\check{V}^{o}_{1,t+1}(\pi) = \max_{c,d}\left\{(1-\beta)u(c) + \beta\check{V}_{2,t+2}\left(d\right)\right\} \\ s.t. \quad d \leq \check{\pi} - \epsilon_{t+1}c,$$

where $\check{\pi}$ is the profit if the agent defaults. As before, we have that:

$$c_{1,t+2} = \frac{(1-\beta)\check{\pi}}{\epsilon^{n}_{t+1}(1-\mu^{n}_{t+1})}, \qquad d_{1,t+1} = \beta\check{\pi}$$

And so we have:

$$\begin{aligned}\check{V}^n_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\check{\pi}}{\epsilon^n_{t+1}(1-\mu^n_{t+1})}\right) + \beta\log\left(\check{\nu}_{2,t+2}\beta\check{\pi}\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^n_{t+2}(1-\mu^n_{t+1})\right) + \log(\check{\pi})\end{aligned}$$

Now consider the problem of an agent at age 0 during the morning market. If they choose $n_1 = o$ intending to repay, then the same as before:

$$\begin{aligned} V_{0,t} &= \mathbb{E}_t\left[\max_{n_0,x,\pi}\left\{\zeta^{n_0}_{0,t} + \zeta^o_{1,t+1} + V^o_{1,t+1}(\pi)\right\}\right] \quad s.t. \\ \pi &= \epsilon^o_{t+1}(1-\mu^o_{t+1})zx^{\alpha} - R^{bn_0}_{t,t+1}x \end{aligned} \tag{B.10}$$

For a given choice of n_0, they choose x to maximize:

$$\max_x\left\{\epsilon^{n_1}_{t+1}(1-\mu^o_{t+1})zx^{\alpha} - \epsilon^{n_0}R^{bn_0}_{t,t+1}x\right\}$$

Taking the FOC gives that producer labor demand, output, and profit are given by:

$$x_{0,t} = \left(\frac{\alpha z(1-\mu^{n_0}_{t+1})}{R^{bn_0}_{t,t+1}}\right)^{\frac{1}{1-\alpha}}, \qquad y^{n_0}_{1,t+1} = z\left(\frac{\alpha z(1-\mu^o_{t+1})}{R^{bn_0}_{t,t+1}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t+1} = \left(\frac{\epsilon^o_{t+1}\alpha z(1-\mu^o_{t+1})}{(R^{bn_0}_{t,t+1})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

Returning to the value functions, we have that:

$$\begin{aligned}V^o_{1,t+1}(\pi) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^o_{t+1}(1-\mu^o_{t+1})}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\beta\pi\right) \\ &= \log\left((1-\beta)^{1-\beta}\beta^{\beta}\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^o_{t+2}(1-\mu^o_{t+1})\right) + \log(\pi)\end{aligned}$$

If they choose $n_1 = o$ intending to default, then they choose where to purchase input goods, where to sell output goods, and the quantity of input goods to solve (B.11) below:

$$\begin{aligned} \check{V}_{0,t} &= \mathbb{E}_t\left[\max_{n_0,x,\pi}\left\{\zeta^{n_0}_{0,t} + \zeta^o_{1,t+1} + \check{V}^o_{1,t+1}(\check{\pi})\right\}\right] \quad s.t. \\ \check{\pi} &= \epsilon^{n_1}_{t+1}(1-\mu^o_{t+1})zx^{\alpha} - R^{bn_0}_{t,t+1}x \end{aligned} \tag{B.11}$$

For a given choice of n_0, they choose x to maximize:

$$\max_x \left\{\epsilon^o_{t+1}(1-\mu^o_{t+1})zx^\alpha - \epsilon^{n_0}\chi x\right\}$$

Taking the FOC gives producer labor demand, output, and profit to be:

$$x_{0,t} = \left(\frac{\alpha z(1-\mu^o_{t+1})}{\epsilon^{n_0}\chi}\right)^{\frac{1}{1-\alpha}}, \qquad y^{n_0}_{1,t+1} = z\left(\frac{\alpha z(1-\mu^o_{t+1})}{\epsilon^{n_0}\chi}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\check{\pi}_{1,t+1} = \left(\frac{\epsilon^o_{t+1}\alpha z(1-\mu^o_{t+1})}{(\epsilon^{n_0}\chi)^\alpha}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right).$$

So, their value is:

$$\begin{aligned}\check{V}^o_{1,t+1}(\check{\pi}) &= (1-\beta)\log\left(\frac{(1-\beta)\pi}{\epsilon^o_{t+1}(1-\mu^o_{t+1})}\right) + \beta\log\left(\check{\nu}_{2,t+2}\beta\check{\pi}\right)\\ &= \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^o_{t+2}(1-\mu^o_{t+1})\right) + \log(\check{\pi})\end{aligned}$$

<u>Incentive compatibility:</u> So, for a given n_0, the agent chooses to not default if:

$$V^o_{1,t+1}(\pi) \geq \check{V}^o_{1,t+1}(\check{\pi})$$

which is equivalent to:

$$\begin{aligned}&\log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\bar{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^o_{t+2}(1-\mu^o_{t+1})\right) + \log(\pi)\\ &\geq \log\left((1-\beta)^{1-\beta}\beta^\beta\right) + \beta\log\left(\check{\nu}_{2,t+2}\right) - (1-\beta)\log\left(\epsilon^o_{t+2}(1-\mu^o_{t+1})\right) + \log(\check{\pi})\end{aligned}$$

and so:

$$\begin{aligned}\log\left(\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}}\right) + \log\left(\frac{\pi}{\check{\pi}}\right) &\geq 0\\ \Leftrightarrow \frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} &\geq \frac{\check{\pi}}{\pi}\end{aligned}$$

where:

$$
\begin{aligned}
\frac{\bar{\nu}_{2,t+2}}{\check{\nu}_{2,t+2}} &= \left(\frac{R_{t+1,t+2}/\epsilon_{t+2}}{\sum_{n_2}\left(\zeta_{t+2}^{n_2}R_{t+1,t+2}^{n_2}/\epsilon_{t+2}^{n_2}\right)^{\gamma_2}}\right)^{\beta\gamma_2} \\
\frac{\check{\pi}}{\pi} &= \left(\frac{R_{t,t+1}^{bn_0}}{\chi}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

□

B.3 Platform Problem

Substituting the equilibrium conditions into the profit function:

$$
\begin{aligned}
\pi_t^s &= \mu_t \eta_{1,t}^p \sum_{n_0} \eta_{0,t-1}^{n_0} y_{1,t}^{(n_0,p)} \\
&= \mu_t \left(\frac{1}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}}\right) \\
&\qquad \times \sum_{n_0}\left(\frac{1}{1+\left(\frac{\zeta^{n_0'}}{\zeta^{n_0}}\right)^{\gamma_0}\left(\frac{\epsilon_t^{n_0}((1-\kappa)R_{t-1,t}^b+\kappa R_{t-1,t}^{n_0'})}{\epsilon_t^{n_0'}((1-\kappa)R_{t-1,t}^b+\kappa R_{t-1,t}^{n_0})}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}\right) \\
&\qquad \times z\left(\frac{\alpha z(1-\mu_t)}{\frac{\epsilon_{t-1}^{n_0}R_{t-2,t-1}^b}{(1-\kappa)R_{t-2,t-1}^b+\kappa R_{t-2,t-1}^{n_0}}R_{t-1,t}^b}\right)^{\frac{\alpha}{1-\alpha}} \\
&= z\left(\frac{\alpha z}{R_{t-1,t}^b}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu_t(1-\mu_t)^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}}\right) \\
&\qquad \times \sum_{n_0}\left(\frac{\left(\frac{(1-\kappa)R_{t-2,t-1}^b+\kappa R_{t-2,t-1}^{n_0}}{\epsilon_{t-1}^{n_0}R_{t-2,t-1}^b}\right)^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{\zeta^{n_0'}}{\zeta^{n_0}}\right)^{\gamma_0}\left(\frac{\epsilon_t^{n_0}((1-\kappa)R_{t-1,t}^b+\kappa R_{t-1,t}^{n_0'})}{\epsilon_t^{n_0'}((1-\kappa)R_{t-1,t}^b+\kappa R_{t-1,t}^{n_0})}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}\right)
\end{aligned}
$$

which can be expanded as:

$$\pi_t^s = z\left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu_t(1-\mu_t)^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}}\right)$$
$$\times\left(\frac{\left(\frac{(1-\kappa)R^b_{t-2,t-1}+\kappa R^{\$}_{t-2,t-1}}{\epsilon_{t-1}R^b_{t-2,t-1}}\right)^{\frac{\alpha}{1-\alpha}}}{1+(\zeta)^{\gamma_0}\left(\frac{\epsilon_t((1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t})}{R^b_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}+\frac{1}{1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{R^b_{t-1,t}}{\epsilon_t((1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t})}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}\right)$$

where ϵ_t is given by:

$$\epsilon_t = \left[\frac{(\zeta)^{\gamma_1-\gamma_2}}{(1-\mu_t)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}}\left(\frac{(1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t}}{R^b_{t-1,t}}\right)^{1+\gamma_2}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

We denote:

$$\begin{aligned}\pi_t^s &= \pi^s(\mu_t,\epsilon_t,\epsilon_{t-1},R^b_{t-1,t},R^b_{t-2,t-1},R^{\$}_{t-1,t},R^{\$}_{t-2,t-1})\\ \epsilon_t &= \epsilon(\mu_t,R^b_{t-1,t},R^{\$}_{t-1,t}).\end{aligned}$$

The Lagrangian for the platform is:

$$\mathcal{L} = \sum_{t=0}^{\infty}\hat{\xi}_{0,t}\pi_t^s(\mu_t,\epsilon_t,\epsilon_{t-1},R^b_{t-1,t},R^b_{t-2,t-1},R^{\$}_{t-1,t},R^{\$}_{t-2,t-1})$$

The FOC for μ_t is given by:

$$\begin{aligned}0 = {}&\hat{\xi}_{0,t}\frac{\partial\pi_t^s(\mu_t,\epsilon_t,\epsilon_{t-1},R^b_{t-1,t},R^b_{t-2,t-1},R^{\$}_{t-1,t},R^{\$}_{t-2,t-1})}{\partial\mu_t}\\ &+\hat{\xi}_{0,t}\frac{\partial\pi_t^s(\mu_t,\epsilon_t,\epsilon_{t-1},R^b_{t-1,t},R^b_{t-2,t-1},R^{\$}_{t-1,t},R^{\$}_{t-2,t-1})}{\partial\epsilon_t}\frac{\partial\epsilon(\mu_t,R^b_{t-1,t},R^{\$}_{t-1,t})}{\partial\mu_t}\\ &+\hat{\xi}_{0,t+1}\frac{\partial\pi_{t+1}^s(\mu_{t+1},\epsilon_{t+1},\epsilon_t,R^b_{t,t+1},R^b_{t-1,t},R^{\$}_{t,t+1},R^{\$}_{t-1,t})}{\partial\epsilon_t}\frac{\partial\epsilon(\mu_t,R^b_{t-1,t},R^{\$}_{t-1,t})}{\partial\mu_t}\end{aligned}$$

where:

$$\frac{\partial \pi_t^s(\mu_t, \epsilon_t, \epsilon_{t-1}, R^b_{t-1,t}, R^b_{t-2,t-1}, R^{\$}_{t-1,t}, R^{\$}_{t-2,t-1})}{\partial \mu_t}$$

$$= z\left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}} \left[\frac{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}\right)\left((1-\mu_t)^{\frac{\alpha}{1-\alpha}}-\mu_t\frac{\alpha}{1-\alpha}(1-\mu_t)^{\frac{\alpha}{1-\alpha}-1}\right)}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}\right)^2}\right.$$

$$\left.-\frac{\mu_t(1-\mu_t)^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1\left(\frac{1}{\zeta}\right)^{\gamma_1}(\epsilon_t)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}(1-\mu_t)^{-\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1-1}}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}\right)^2}\right]$$

$$\times\sum_{n_0}\left(\frac{z\left(\frac{(1-\kappa)R^b_{t-2,t-1}+\kappa R^{n_0}_{t-2,t-1}}{\epsilon^{n_0}_{t-1}R^b_{t-2,t-1}}\right)^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{\zeta^{n_0'}}{\zeta^{n_0}}\right)^{\gamma_0}\left(\frac{\epsilon_t^{n_0}((1-\kappa)R^b_{t-1,t}+\kappa R^{n_0'}_{t-1,t})}{\epsilon_t^{n_0'}((1-\kappa)R^b_{t-1,t}+\kappa R^{n_0}_{t-1,t})}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}\right)$$

and

$$\frac{\partial \pi_t^s(\mu_t, \epsilon_t, \epsilon_{t-1}, R^b_{t-1,t}, R^b_{t-2,t-1}, R^{\$}_{t-1,t}, R^{\$}_{t-2,t-1})}{\partial \epsilon_t}$$

$$= z\left(\frac{\alpha z}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}\mu_t(1-\mu_t)^{\frac{\alpha}{1-\alpha}}\times\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_t}{1-\mu_t}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}\right)^{-1}$$

$$\times\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1\left(\frac{1}{\zeta}\right)^{\gamma_1}(\epsilon_t)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}(1-\mu_t)^{-\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1-1}$$

$$\times\left[\frac{\left(\frac{(1-\kappa)R^b_{t-2,t-1}+\kappa R^{\$}_{t-2,t-1}}{\epsilon_{t-1}R^b_{t-2,t-1}}\right)^{\frac{\alpha}{1-\alpha}}}{\left(1+(\zeta)^{\gamma_0}\left(\frac{\epsilon_t((1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t})}{R^b_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2}(\zeta)^{\gamma_0}\left(\frac{(1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t}}{R^b_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\frac{\alpha\gamma_0}{1-\alpha}\epsilon_t^{-\frac{\alpha\gamma_0}{1-\alpha}-1}\right.$$

$$\left.-\frac{1}{\left(1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{R^b_{t-1,t}}{\epsilon_t((1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t})}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\right)^2}\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{R^b_{t-1,t}}{(1-\kappa)R^b_{t-1,t}+\kappa R^{\$}_{t-1,t}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}\frac{\alpha\gamma_0}{1-\alpha}\epsilon_t^{\frac{\alpha\gamma_0}{1-\alpha}-1}\right]$$

and

$$\frac{\partial \pi^s_{t+1}(\mu_{t+1}, \epsilon_{t+1}, \epsilon_t, R^b_{t,t+1}, R^b_{t-1,t}, R^\$_{t,t+1}, R^\$_{t-1,t})}{\partial \epsilon_t}$$

$$= z\left(\frac{\alpha z}{R^b_{t,t+1}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\mu_{t+1}(1-\mu_{t+1})^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon_{t+1}}{1-\mu_{t+1}}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}}\right)$$

$$\times \frac{\left(\frac{(1-\kappa)R^b_{t-1,t}+\kappa R^\$_{t-1,t}}{R^b_{t-1,t}}\right)^{\frac{\alpha}{1-\alpha}}\left(-\frac{\alpha}{1-\alpha}\right)\epsilon_t^{-\frac{\alpha}{1-\alpha}}}{1+(\zeta)^{\gamma_0}\left(\frac{\epsilon_{t+1}((1-\kappa)R^b_{t,t+1}+\kappa R^\$_{t,t+1})}{R^b_{t,t+1}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}$$

and

$$\frac{\partial \epsilon(\mu_t, R^b_{t-1,t}, R^\$_{t-1,t})}{\partial \mu_t}$$

$$= \left[(\zeta)^{\gamma_1-\gamma_2}\left(\frac{(1-\kappa)R^b_{t-1,t}+\kappa R^\$_{t-1,t}}{R^b_{t-1,t}}\right)^{1+\gamma_2}\right]^{\left(\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)\right)^{-1}}$$

$$\times \left(-\frac{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}\right)(1-\mu_t)^{-\frac{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}}$$

We now characterize steady state equilibrium with an optimizing platform. We have that:

$$
\begin{aligned}
0 &= \partial_1 \pi^s(\mu, \epsilon, \epsilon, R^b, R^b, R^\$, R^\$) \\
&\quad + \partial_2 \pi^s(\mu, \epsilon, \epsilon, R^b, R^b, R^\$, R^\$)\partial_1 \epsilon(\mu, R^b, R^\$) \\
&\quad + \partial_3 \pi^s(\mu, \epsilon, \epsilon, R^b, R^b, R^\$, R^\$)\partial_1 \epsilon(\mu, R^b, R^\$) \\
\epsilon &= \left[\frac{\zeta^{\gamma_1-\gamma_2}}{\mathcal{S}^{1+\gamma_2}\left(1-\mu\right)^{\frac{\gamma_1+\alpha}{1-\alpha}-\gamma_1(1-\beta)}}\right]^{\frac{1}{\frac{\gamma_1+\alpha}{1-\alpha}+1+\gamma_2-\gamma_1(1-\beta)}} \\
R^b &= \frac{\sum_{n_1}\eta_1^{n_1}\left(\frac{\alpha z(1-\mu^{n_1})\epsilon^{n_1}}{\mathcal{S}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\eta_0^m}{(\epsilon^m)^{\frac{\alpha}{1-\alpha}}}\left(1+\mathcal{S}\right)+\eta_0^p\right)+q^s}{(1-\eta_2^m)\sum_{n_0,n_1}\eta_0^{n_0}\eta_1^{n_1}\beta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha z(1-\mu^{n_1})\epsilon^{n_1}}{\left(\epsilon^{n_0}\frac{R^b}{R^{n_0}}\right)^{\alpha}}\right)^{\frac{1}{1-\alpha}}} \\
R^\$ &= \frac{1}{\mu^M}, \quad \mathcal{S} = \frac{R^b}{(1-\kappa)R^b+\kappa R^\$}, \quad q^s = \frac{\pi^s}{R^b-1} \\
\pi^s &= z\left(\frac{\alpha z}{R^b}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu(1-\mu)^{\frac{\alpha}{1-\alpha}}}{1+\left(\frac{1}{\zeta}\right)^{\gamma_1}\left(\frac{\epsilon}{1-\mu}\right)^{\left(\frac{1}{1-\alpha}+\beta-1\right)\gamma_1}}\right) \\
&\quad \times\left(\frac{(\epsilon\mathcal{S})^{-\frac{\alpha}{1-\alpha}}}{1+(\zeta)^{\gamma_0}\left(\frac{\epsilon}{\mathcal{S}}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}+\frac{1}{1+\left(\frac{1}{\zeta}\right)^{\gamma_0}\left(\frac{\mathcal{S}}{\epsilon}\right)^{-\frac{\alpha\gamma_0}{1-\alpha}}}\right) \\
\eta_0^{n_0} &= \frac{\left(\zeta^{n_0}(R^{bn_0})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}}{\sum_{n_0'}\left(\zeta^{n_0'}(R^{bn_0'})^{-\frac{\alpha}{1-\alpha}}\right)^{\gamma_0}} \\
\eta_1^{n_1} &= \frac{\left(\zeta^{n_1}\left(\epsilon^{n_1}(1-\mu^{n_1})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}}{\sum_{n_0'n_1'}\left(\zeta^{n_1'}\left(\epsilon^{n_1'}(1-\mu^{n_1'})\right)^{\frac{1}{1-\alpha}+\beta-1}\right)^{\gamma_1}} \\
\eta_2^{n_2} &= \frac{\left(\zeta^{n_2}R^{dn_2}/\epsilon^{n_2}\right)^{\gamma_2}}{\sum_{n_2'}\left(\zeta^{n_2'}R^{dn_2'}/\epsilon^{n_2'}\right)^{\gamma_2}} \\
R^{bn} &= \epsilon_t^n\mathcal{S}^n R^b, \quad \mathcal{S}^n = \frac{R^b}{(1-\kappa)R^b+\kappa R^n} \\
R^{dn} &= (1-\kappa)R^b+\kappa R^n = R^b/\mathcal{S}^n
\end{aligned}
$$

with the supplemental equilibrium equations:

$$x_{0,t}^{\mathbf{n}_1} = \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{R_{t,t+1}^{bn_0}}\right)^{\frac{1}{1-\alpha}}, \qquad y_{1,t}^{\mathbf{n}_1} = z\left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{R_{t-1,t}^{bn_0}}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\pi_{1,t}^{\mathbf{n}_1} = \left(\frac{\alpha z(1-\mu_t^{n_1})\epsilon_t^{n_1}}{(R_{t-1,t}^{bn_0})^{\alpha}}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) \qquad c_{1,t}^{\mathbf{n}_1} = \frac{(1-\beta)\pi_{1,t}}{(1-\mu_t^{n_1})\epsilon_t^{n_1}}$$

$$d_{1,t}^{\mathbf{n}_1} = \beta\pi_{1,t}^{\mathbf{n}_1}, \qquad c_{2,t}^{\mathbf{n}_2} = \frac{R_{t-1,t}^{dn_2}\beta\pi_{1,t-1}^{\mathbf{n}_1}}{\epsilon_t^{n_2}}.$$

B.4 Proofs for Platform Competition (Preliminary)

Proof of Proposition 1. (i) The market equilibrium is the same as in subsection 3.3 except that now $(1-\mu_t)$ is replaced by $(1-\mu_t^1)/(1-\mu_t^2)$. If χ is sufficiently large that the threat of exclusion from either platform is sufficient to incentive funds to repay loans on that ledger, then $q^{E\mathcal{L}} = \tilde{q}^{E\mathcal{L}}$ and there is no need to bargain over enforcement because it doesn't require cooperation. If χ is sufficiently low that only exclusion from both platforms is sufficient to incentivize repayment, then for both $\mathcal{L}$, we have $q^{E\mathcal{L}} = q^E$ and $\tilde{q}^{E\mathcal{L}} = 0$ so outcome of the Nash Bargaining is cooperation on enforcement without a transfer $T = 0$.
(ii) For ζ close to 1, when the trading advantage of platform 1 is not too large, the possible outcomes look like those in subsection 3.3. That is, if χ is large, then both platforms are able to enforce contract without cooperation and if χ is small, then cooperation is required for any contract enforcement. However, when χ and ζ are large, it is possible that, under non-cooperation, platform 1 can enforce contracts while platform 2 cannot. In this case, ledger 1 becomes the dominant ledger and so the currency market frictions become relevant.

Platform bargaining at $t = 0$ is now more complicated because the outside option for platform 1 is more complicated. If χ and ζ are sufficiently large that ledger 1 can incentivize contract enforcement on their ledger without cooperation and $\varsigma\epsilon < 1$, then $\tilde{q}^{E1} > q^{E1}$ and so ledger 1 prefers the non-cooperative outcome. This means that the transfer platform 2 would have to pay to get enforcement leads to negatives surplus:

$$q^{E0} - \tilde{q}^{E2} - T = q^{E1} - \tilde{q}^{E1} < 0$$

and so the bargaining breaks down.

$\square$